DIPLOMA IN ELEMENTARY EDUCATION (D.El.Ed.)

Course-504

Learning Mathematics at Elementary Level

Block -1

Importance of Learning Mathematics at the Elementary Stage of Schooling

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Credit points (4=3+1)

Block	Unit	Name of Unit	Theory Study Hours		Practical Study
			Content	Activity	
Block-1: Importance of Learning Mathematics at the Elementary Stage of Schooling	**U1**	How children learn mathematics	3	2	Seminar on mathematics is for all, mathematics phobia
	U2	Mathematics and Mathematics Education - Importance, Scope and Relevance	4	2	-
	U3	Goals and Vision of Mathematics Education	4	2	Taking mathematics learning beyond classroom Identification of problems in mathematics education in your class
	U4	Learner and Learning – centered methodologies	5	3	Organizing mathematics club in your school
Block 2: Enriching Contents and Methodology	**U5**	Numbers, Operations on Numbers	5	2	-
	U6	Shapes and Spatial Relationships	5	2	-
	U7	Measures and Measurements	4	2	-
	U8	Data Handling	4	3	Statistical analysis of data
	U9	Algebra as generalized Arithmetic	4	2	-
Block 3: Learner Assessment in Mathematics	**U10**	Approaches to Assessment of Learning Mathematics	3	2	Development of a lesson plans and preparation of concept maps in mathematics
	U11	Tools and Techniques of Assessment	4	3	Development of exhibits for mathematics laboratory
	U12	Follow up of Assessment of Learning Mathematics	3	2	Identification of problems and preparation of remedial measures in learning mathematics
		Tutoring	15		
		Total	63	27	30
Grand Total			**63+27+30=120 hrs.**		

Block 1

Importance of Learning Mathematics at the Elementary Stage of Schooling

Block Units

Unit 1 How children learn mathematics

Unit 2 Mathematics and Mathematics Education – Importance, Scope and Relevance

Unit 3 Goals and Vision of Mathematics Education

Unit 4 Learner and Learning – centered methodologies

BLOCK INTRODUCTION

You as a learner will study Block 1:

Importance of learning mathematics at the Elementary stage of Schooling. This Block consists four units relating to importance of learning mathematics. Every units is divided into sections and subsections.

UNIT-1 This unit will empower you to understand how children learn mathematics actually. How does a child think and what are the Stages involved in cognitive development? There is a close relationship between the growth of thinking and the development of mathematical concepts. As a teacher, one should know how to deal with mathematics phobia which is present in students and making mathematics learning more pleasurable.

UNIT-2 This unit will empower you to observe the nature of mathematics on the basis of which mathematics education can be designed by teachers for learners. The importance of mathematics education can also be realized in the context that how mathematics will help in real life situations and develop attitude of Problem sowing?

UNIT-3 You will be acquainted with the aims of Mathematics Education. There will be can acquaintance with the methods of imparting Mathematics education beyond classroom and making it more Joyful so that Phobia for mathematics can be easily removed.

UNIT-4 This unit will empower you to understand the methods for teaching and learning mathematics like inductive and deductive, Analysis and synthesis methods, Project and Problem solving method. The rote learning not only makes understanding of mathematical concepts more difficult but increase fear for mathematics which inhibits further learning of the subject, so there is need of learning centered Approaches of Teaching mathematics like concept mapping, Activity based approach. These approaches are followed to develop learner's creative abilities and there is proper focus on use of mathematics laboratory and library. There are Some emerging trends in learning mathematics like cognitive, constructivist and experiential approach.

CONTENTS

UNIT 1 HOW CHILDREN LEARN MATHEMATICS

Structure

1.0 INTRODUCTION

Among all the school subjects, maximum emphasis is attached to learning mathematics. You must have experienced, during your student days and also being a teacher, the extent of pressure exerted on children to perform at a higher level in mathematics in comparison with other subjects. Even the parents, irrespective of their educational status, press their children very hard to perform well in mathematics and perhaps maximum study time at home is spent on doing homework in mathematics than in any other subject. In almost all cases, the condition of the learner for learning mathematics is rarely taken into consideration. There is a general feeling that the child is a mini adult and that he/she can acquire the mathematical skills effectively, either through developing understanding or through rote memory, very often through latter. Driven by such belief, the parents and teachers insist on rote memorization of a lot of mathematical concepts, facts and tables. The result is that most of the children being subjected to mechanical memorization without real understanding of principles and processes begin to develop phobia for mathematics from the beginning of their school learning which may aggravate and even continue throughout their life.

You might have experienced that the mathematical concepts and processes at different levels, especially at the primary level, are arranged from simpler to complex order. Have you ever thought that such arrangement has anything to do with the growth and development of learner, particularly whether these are associated with the development of their thinking abilities? Research has established a close relationship between the growth of thinking and the development of mathematical concepts. As a teacher you should be aware of such relationship so that you can develop understanding of the strength and difficulties of every child in your class in their learning of mathematics concepts and can take appropriate facilitating steps in that direction.

When the learners' needs and interests are not properly understood and facilitated, and on the other hand they are forced to learn mathematics literally as prescribed in the textbooks, the learning of mathematics becomes a burden and problematic as is being experienced in majority of children. They develop anxiety to perform which in the long run creates phobia for mathematics, difficult to get rid of in a large number of cases. If, as teachers, we have clear vision of how children learn mathematics in a joyful manner, we can be able to properly facilitate their learning mathematics from the beginning days of their school learning.

In this unit i.e., the first unit of this course on mathematics learning, we have tried to discuss the ways the children love to learn mathematics by studying the development of the mathematical concepts in accordance with the cognitive development of the child. We have also tried to search for the typical problems of children in learning mathematics which lead to the development of fear for mathematics and the ways to make mathematics learning a pleasant experience for them.

For completing the study of this unit you will need approximately *06 (six) hours*.

1.1 LEARNING OBJECTIVES

After studying this unit, you will be able to

- recognise the developmental trends in learning mathematics concept during early childhood.
- explore the ways to facilitate children's learning of mathematics at different stages of development.
- ascertain the difficulties faced by children in learning mathematics at the early stage of learning and devise methods to make mathematics learning pleasurable by overcoming those difficulties.

1.2 THE WAYS A CHILD THINKS

You come across large number of children every day, in your family, in the school, in the market, on the road side and everywhere around you. You must be interacting with them on innumerable occasions every day. In course of your interactions with a child,

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what is your feeling about the child, the way he/she thinks and the way he/she learns? Is he/she like any other adult person so far as the ways of thinking and learning are concerned? Do you think the child begins to think and learn when he/she in the school? Here are some beliefs about the child and the ways a child thinks:

"A child's mind is like a clean slate to be written upon."

"A child's mind is in totally dark which is to be illuminated by knowledge."

"A child is like a lump of clay which can be given any shape as desired."

"A child is like a green plant who should be nurtured."

"A child's mind is like an empty pot to be filled in by knowledge."

Which of these statements do you believe to be appropriately describing the child's mind? It is difficult to know what type of thinking is there in the mind of any individual, irrespective of his/her age. What we are interested from teaching-learning point of view is not exactly what is in the mind of the child but how he/she is using the mind or more specifically how he/she is thinking.

The base of thinking is perception, and perception comes from observing, experiencing and interacting with objects in the environment. The young child's first interactions with his/her environment are based almost totally on the sense experiences: mostly by seeing, and touching and sometimes by hearing and also by tasting. Many psychologists, prominent among them are Piaget, and Bruner, believe that perceiving through manipulation of 'concrete' objects forms the basis of human knowledge and thinking. Piaget, the famous Swiss child psychologist, proposed that child's thinking begins with two processes: *perception* (the knowledge of objects resulting from direct contact with them) and *representation* (mental imagery of the perceived objects). Of course, for giving shape to the representation, language plays an important part.

As teachers, we have to keep in mind some principles of perception so as to facilitate children's thinking process without creating any hindrances. Some of the important principles are as follows which emerged from the studies of Adelbert Ames Jrfar back in1938 and are getting much attention in recent times:

> We do not get our perceptions from the objects around us. Our perceptions come from us. It is not to belittle the importance of the object but to highlight the way in which each observer perceives the object. Numbers are quite fascinating to many while these are dreaded figures for quite a large proportion of people including adults and children.
>
> What we perceive is largely a function of our previous experiences, our assumptions and our purposes or needs.
>
> We are unlikely to alter our perceptions until and unless we are frustrated in our attempts to do something based on them. If suppose a child has observed her mother combining two balls of dough into one and preparing a bread of it, she perceives it as the addition to be 1+1=2. She is not likely to change her perception until she has to add one solid object (like a marble) with another of the kind.

Since our perceptions come from us and our past experiences, it is obvious that each individual will perceive the same object in a unique way. Communication is possible only to the extent that two perceivers have similar purposes, assumptions and experience.

The meaning of a perception is how it causes us to *act*. When it rains, some people run for shelter and some enjoy dancing in the rain. Their perceptions of what is happening are different as reflected in their actions.

Representation is the process of formation of images of the objects when the object is not in the field of direct observation. In this situation, the child requires to describe the object that is in his/her mind using language in some form or other. Therefore, language is called the vehicle of thought.

ACTIVITY- 1

Give the name of any one object (say a pencil). Ask the students to tell whatever that comes to mind immediately after hearing the name of the object (each has to tell only one such). List down the responses of students and observe the multifarious connotations given by the students.

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Piaget conceptualized the structure of the process of knowing or cognition which he considered invariant in all situations and for all individuals. He stipulated that every child has a mental structure which is activated when it comes across an object, a process or an event. The child tries the twin processes of **assimilation** (interpreting the perceived object, process or event with the existing mental structure) and **accommodation** (modifying the existing structure to interpret the object, event or process). When the child strikes a balance between the two processes while trying to internalize the perceived object, **adaptation** (a relatively stable structure) takes place. The process of striking a balance between accommodation and assimilation is called **equilibration** which is very important in Piaget's theory of cognitive development. Although, each individual's thought process follows the invariant process of organisation and adaptation as equilibration of assimilation and accommodation, yet everyone is unique in his/her ways of thinking. That is because the individuals differ in their perception and representation, in the ways of processing their experiences through equilibrating their assimilation, accommodation and finally in organising the adapted bits of thinking. In each of the processes and at each stage of thinking, each child has its own and unique way.

Let us have a brief overview of the stages of cognitive development of children as stated by Piaget which has been now accepted universally. This will provide us with insight into the ways a child develops his/her thinking as he/she grows up. This will be of particular interest for us teaching mathematics, because Piaget has elaborated the stages with more experimentation in mathematical concepts.

E1. What are the two basic processes of development of thinking? Explain each of them by an example.

E2. The equilibration or maintaining a balance between which two processes constitute the basic functioning of thinking?

1.2.1 Stages of Cognitive Development

Not only are children's ways of thinking different than adults, you must have experienced that children of different age groups demonstrate different patterns in their ways of thinking. That is precisely the observations of Piaget while developing his stage theory of cognitive development in children. Piaget observed his own three children very minutely from their birth and grouped the children's actions (more specifically 'operations') according to some similarities and came out with the *stage* or *period* specific patterns of actions and demonstrated that children follow certain broad stages or period of development of thinking or cognition.

Accordingly, the stages or periods of cognitive development have been categorised by Piaget as:

Sensory-motor Period (from birth to 2 years),

Pre-operation Period (from 2 to 7 years),

Concrete Operation Period (7 to 11 years), and

Formal Operation Period (11-12 years to 14-15years).

Sensory-motor Period: The first stage, from birth to one-half or two years, is a pre-verbal, pre-symbolic period categorised by direct actions like sucking, looking, grasping etc which are at first un-coordinated and then gradually become coordinated. There is a progression from spontaneous movements and reflexes to acquired habits and from these to intelligent activities. As for example, one of the first actions the baby displays is that of thumb sucking which is not a reflex action but a habit that the child discovers and finds satisfactory. Such habits may grow out of child's different reflex actions or may be the results of conditioning imposed from outside.

At about one year of age, there is a new element in the child's behaviour. He/she can establish an objective or intention in his/her actions. In order to get a ball placed at a distance on the rug be is lying that is out of reach, he can develop a procedure for obtaining the ball. He may discover that moving the rug moves the ball, so he pulls the rug (and the ball) towards himself. Such actions display the intentionality behind the act and Piaget considered it as an intelligent act. The child is beginning to think with some purpose or an end and then a search for appropriate means to that end.

Further, towards the end of this period, the child begins to talk using mono-syllables of the language used at home. This is indicative of onset of symbolic action, a component of intelligence.

Pre-operation Period: This period beginning from one-half or two years lasts until seven years of age roughly coincides with preschool age. This is characterised as the stage of **representation** or **symbolism**. Among symbolic functions are languages, symbolic plays, the invention of fictions, and deferred imitations. During the sensory motor period, there is no use of words or symbols to represent things, and imaginations. But in pre- operation period, the girl child uses words to indicate objects or actions, involves himself/herself in playing "let's pretend" games which are symbolic in nature involving imitation of real life characters. In deferred imitation, the child engages herself in playing activities like cooking, dressing toys, modeling or drawing or in such other activity which requires imitation of the model when it is no longer present. Through such activities, representing formation is possible. Representation formation is the transformation of actions into thoughts, thus internalising overt activity which facilitates in expanding the dimensions of thought.

The preoperational thought period is devoid of reversible operations and concept of conservation. Children of four to six years of age can pour liquid from a short wide bottle into a taller thin one with the impression that they have more liquid in the taller container. Even reversing the process does not satisfy them that quantity remains the same despite the one bottle being taller than the other.

Concrete Operation Period: The third stage, from approximately seven to eleven or twelve years of age, is that of concrete operations. It is particularly important for you, because most of the time that children are in the elementary school they are in this stage of development.

This stage marks the beginning of logico-mathematical thought and hence has importance for learning mathematics which we shall be discussing in the subsequent sections in greater detail. The child in this period begins to demonstrate actions (Piaget terms these as 'Operations') indicative of the ability to think logically by physical manipulation of concrete objects. The child is no longer dependent upon perceptions or sensory cues. During this period, the child demonstrates the two major operations of grouping and conservation which are very much associated with the development of mathematical concepts as would be clear from the discussions of the next section.

Formal Operation Period: The fourth stage is the period of formal operations which does not occur until eleven to twelve years of age. The child now in the post-primary grades, reasons hypothetically using symbols or ideas and no more is in need of physical objects as a basis for his/her thinking. The child has attained new mental structures. These new structures include the propositional combinations of symbolic logic like implications (if…then), disjunction (either-or, or both), exclusion (either – or), reciprocal implication and so on.

The child can now understand and carry out calculations involving proportions, which allow him to make maps reduced or enlarged to a desired scale, to solve problems of time and distance, problems of probability and geometrical problems involving similarity.

In brief, the development of thinking or cognition proceeds from the perception and sensory-motor experiences to thinking in terms of manipulating concrete objects and proceeding to think hypothetically and combining several ways of thinking in abstract terms in absence of concrete objects.

Understanding the characteristics of cognitive development will help you to develop insights into the methods of teaching and facilitating learning of mathematics appropriate to the stages of development.

1.2.2 Development of Mathematical Concepts

The three basic groups of mathematical concepts that are essential in all topics included in the mathematics curriculum at the elementary school level are number and operations on numbers, spatial thinking and measurement. The concepts relating to these areas and other areas in elementary school mathematics will be dealt elaborately in the block 2 of this course. In this section, we have tried to point to the developmental aspects of some of the selected concepts associated to the three areas which would provide you to develop insight into the ways you should plan the teaching mathematical concepts to the young children.

Development of Number Concepts: Usually counting is considered as the first step of introducing number concepts. And in most of the children admitted to class I know the number names, at least up to 10 mostly through rote methods. But, for effective learning of number concepts, the children need to acquire some preliminary concepts called the pre-number concepts.

Pre-number concepts: These concepts can be developed in children during the pre-school years. i.e. before attaining 7 years of age(before the concrete operation stage).

Matching: Matching leads to understanding the concept of **one-to-one correspondence**. When a child passes out cookies, each child in the room gets one cookie. Maybe there is just the right amount of cookies or maybe there are extra cookies.

Matching forms the basis for our number system. When a child can create “the same”, it then becomes possible to match two sets. This becomes a pre requisite skill for the more difficult tasks of conservation. When the child places a chocolate for every toy he/she possesses and can emphatically expresses that he/she has a chocolate for every toy or he/she has as many chocolates as the toys, he/she has successfully matched the two groups of materials.

Sorting: Children need to look at the characteristics of different items and find characteristics that are the same.Young children usually begin sorting by color before sorting by other attributes.

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Comparing: Children look at items and compare by understanding difference like big/little, hot/cold, smooth/rough, tall/short, heavy/light. Using comparison terms like these is important when children are looking for a relationship between two or more quantities. To determine more/less/same comparisons, children initially need to construct and compare sets of concrete objects.

At the preschool level children should make comparisons of more, less and same by making visual comparisons.

Ordering: Ordering is foundational to our number system. Children have to be able to put items in an order so they are counted once and only once. Putting items in order is a prerequisite to ordering numbers. **Seriation** is ordering objects by size, length, or height. When giving a child directions use ordinal words (first, next, last)

Subitizing: Instant recognition of a number pattern without counting is subitizing. The pattern can be reconstructed without knowing the amount. Subitizing helps the children see small collections as one unit. This provides an early perceptual basis for number, but it is not yet "number knowledge".

Number Concepts: Counting, recognising and using numerals correctly, comparing numbers and operations on numbers are considered as important milestones in the development of number concepts.

Counting: The common use of numbers is counting. The process of counting involves two steps – first assigning a number to a particular object which forms one of a sequence of objects is known as *ordinal* aspect of number. Second and final step of counting is of knowing that the number of objects in a collection ('manyness', or 'numerosity') i.e. the *cardinal* aspect. Ordinality refers to the order (what is the position?) of the objects in a collection whereas cardinality refers to the size (How many objects?) of the collection. Ordinality develops earlier at about 3 to 5years of age after the child is able to match the objects in two collections using the method of one-to-one correspondence as discussed earlier along with the knowledge of number names. The knowledge of names of numbers from 1 to 9 develops around 2 to 3 years of age not as a matter of numerical ability, but as words in course of language development.Associating these words (one, two, three, four,….., nine) with the objects is the beginning of the development of numerical skill. Establishing one-to-one correspondence between the objects and the number names in a sequential order is the process of ordinality. However, ordinality does not ensure knowing the size of the collection of objects for two reasons – (i) child at the age of 2 to 4 years is yet to know the quantity associated with number names, and (ii) he has not yet developed the conservation of numbers.

For example, the child below 5 years of age would say that there are more objects in the second line than in the first line (Fig.1.1). Here, the child lacks the

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sense that a collection of objects when spread linearly without changing the number remains invariant. When the child acquires this sense of invariance, he/she is said to have acquired the conservation of numbers which comes around age 6.

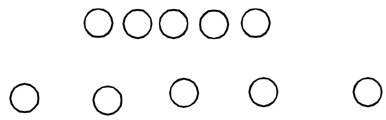

Fig. 1.1

Similarly, the child acquires the conservation of length, area, volume and mass in later years during the concrete operation period (i.e. from age 7to 11 or 12). Once the conservation of numbers is achieved around the age of 4 to 5, he/she can group the objects and count to know the quantity of objects within the group.

Use of numerals: Numerals are the symbols used to represent numbers like 1, 2, 3,… are used for the numbers one, two, three, … and so on. While the concept of numbers is learnt through manipulation of objects and interactions with others, one has to introduce the child about the structure of numerals used for different numbers. In the decimal system of numbers, once the child becomes familiar with the numerals for the single digit numbers from 0 to 9, numerals for other numbers can be constructed by the child. The child becomes ready to understand and use the numerals by the age 7, but it is by the age 11 that he/she can write large numbers stating the place value of each number.

In writing numerals for numbers ten and more than ten, the knowledge of place value is essential which is developed around age 7 to 8 through process of counting in groups which has been dealt in detail later in this course. Once the knowledge of place value is developed, it becomes easier for the child to compare the numbers.

Operations on numbers: Addition and its reverse process subtraction are observed to be performed by children at a very early age even before age 6. Adding and taking away with concrete objects are quite familiar to those who never visit the schools. But the real understanding of the structure and operations comes around the age 9 to 11.

From a developmental standpoint, children are able to learn multiplication at the same time that they are able to learn addition. But in schools it is delayed and multiplication along with division is taught in grade III i.e. till the children attain age 9. Again the structural properties of multiplication and division in natural numbers are introduced in the later part of the concrete operation period around age 11.

***Development of Measurement Concepts*:** The work of Piaget has made a significant contribution to our understanding of the development of measurement concepts in the child. Piaget identifies two processes i.e. conservation and transitivity upon which the measurement process is dependent. We have already discussed the notion of conservation previously in this unit.

The notion of transitivity is best illustrated by an example. Suppose a child is shown a rectangular plot of land in the school garden and was asked to create another such plot in the garden with equal measures of the sides of the given plot. Suppose the length of the given plot is say A. Then the child measured the length to be B by using a measuring stick. Then, the child carved out a plot the measure of which length was C. If he has performed the process of measurement of length correctly, we have a situation where he is displaying his grasp of the fact that if A = B and B = C, then A = C i.e. his plot is of same length as the given one by virtue of the intermediary B (the measurement on the stick) as a comparison. Whatever the measuring situation, the meaningful use of an instrument of measure rests on the notion of transitivity.

Most of the research concerning the development of measurement concepts emanates from Piagetian studies and relates mainly to the measurement of spatial entities such as length.

Initially, the young child in the preschool i.e. below 6 years of age displays no grasp of conservation of length. His judgments are based primarily on a single perceptual feature. At this age a child judges the two lines (Fig.1.2) to be unequal because their end points are not aligned.

Fig. 1.2

Area and volume judgments are usually based on the longest linear dimension (it is bigger because it is longer) as visually perceived by the child.

Around 6 to 7 years of age, the child uses non-standard unit of measuring length like his hand span or his own height to measure the length

The child begins to understand the conservation of volume of liquid at about 7 to 8 years of age when he/she realizes that the liquid poured from a wider vessel from a tall thin vessel is of same amount.

It is not until roughly around 8 to 10 years of age the average child can appreciate measurement in terms of covering whatever is to be measured, with smaller units of measure. Up to this stage, development of the measurement has been characterised by a trial and error approach. Now the child is able to proceed by means of a more calculated approach. However, the measurements of area and volume in terms of space occupied by a particular object lags behind.

The child reaches the final stage of development to measure area and volume by calculation of linear dimensions (length, breadth, and height/thickness) by the age of 10 and 11 or a few years later.

***Development of Spatial Thinking*:** The child's first impression of space or world in which that lives is a very disorganized one. Neither can she/he discriminate the figures nor can he hold the image of the figure for long time. When the child has passed the scribbling stage, around three and half years of age, he/she can distinguish between the closed and open figures. But all simple closed figures such as squares, circles or triangles are all the same for him/her and are drawn the same way.

Around 7 to 8 years of age the child becomes able to differentiate between similar shapes such as squares, rectangles and rhombus correctly. But it is not until the child reaches 10 years old, he/she is able to name the figures correctly and it is not until a year or two older than this that he/she could distinguish the presentation of 3D objects from those of the 2D figures.

There are more intricate aspects of development of spatial thinking which mostly unfold during the later years of the concrete operation period and mostly during the formal operation period.

E3. Which of the pre-number concepts are used for classification of objects?

E4. During which of the four stages of cognitive development, most of the mathematical concepts are likely to develop?

E5. In which stage of cognitive development, abstract mathematical concepts are likely to develop?

E6. What is meant by conservation of length?

1.3 MATHEMATICS LEARNING DURING EARLY CHILDHOOD

From the above discussions on the development of mathematical concepts, you can have a lot of ideas about how to facilitate children's learning of mathematics concepts at the early stage of their development and also at the early stage of schooling. We try to present, in this section, the ways of learning mathematics, the forms and causes of fear for mathematics and the ways to make mathematics learning more pleasurable.

1.3.1 Ways of Learning Mathematics

There is no single and definite way of learning mathematics even at the earliest stage of learning. From the previous discussions in this unit, you might have formed some ideas as to the nature of learning mathematics at the early days of schooling. In addition to those, here are some points regarding the characteristics of nature of mathematics learning as summarised by White Bread (Anghileri, 1995):

- Mathematics starts from 'home learning' established in the child before he/she comes to school.
- Mathematics is based on understanding.
- Mathematics puts great emphasis on the child's own methods of calculating and solving problems and rejects the previous practice of heavy emphasis on standard written algorithms.
- Mathematics is regarded as a powerful tool for interpreting the world and therefore should be rooted in real experience across the whole curriculum.
- Mathematics is brought out of the child's everyday situations.
- Mathematics with reason is rooted in action – learning through doing.
- Mathematics with reason puts less emphasis on representing numbers on paper as 'sums' and more emphasis on developing mental images in the child.
- The main tool for child and teacher to employ in the mastery of mathematics concepts is language, not pencil and paper exercises from textbooks. The child is encouraged to talk about what he/she is doing.
- Errors are accepted as essential part of the mathematics learning process. The child, freed from the fear of criticism, will more readily experiment.

From the preceding discussions, we have tried to focus on some basic ways related to the learning of mathematics during the early stages of school learning. These are the ways of learning which you can facilitate in the classroom situations.

Manipulation of objects: As you have observed, it is through the manipulation of concrete objects that the children acquire the mathematical skills at the early stage. Acquisition of any mathematical skill at the early stage like comparison, categorization, counting, fundamental four operations,cannot be possible without manipulation of concrete objects. Provision of a variety of objects both familiar and novel, should be made available to children in the classroom for their free handling, so that it would be easier for you to facilitate their learning of desired mathematical concepts.

Placing tasks in meaningful contexts: In real situations, where the mathematics serves real purposes, young children quickly and easily develop their own informal and largely effective methods. The difficulties start when they enter the school and are expected to operate in the abstract, to use formal 'pencil and paper' routines and procedures and to do mathematics for no clear purpose. Evidence from research about the ways children learn seem to suggest that what we need to do is to start with real problems, and work from them to abstract representations.

There are abundant opportunities in the everyday activities of young children to get themselves involved in rear mathematics. Playing games, sharing sweets, groupings in the class for performing different activities, finding out the number of days,it is to next school holiday are just a few such examples.

With young children in particular, problems can be real yet essentially born out of imagination. Problems arising through imaginative stories and plays can often be even more vivid for young children than genuinely real life problems. Use of fairy tales, adventure stories, comic strips are some of the exemplars of materials that can fire imagination in children.

While young children may be helped to develop their mathematical abilities and understandings by tackling real problems placed within contexts which are meaningful to them, it is important that they learn to depend less upon the support on such contexts. The same process or concept needs to be presented to them in a variety of meaningful contexts. In this way, by natural processes of induction, children are able to sort out the relevant from the irrelevant and they are ultimately able to abstract for themselves the essential elements of the process or concept. All the while we must keep in mind that mathematics gains its power from abstractness, and children need to be helped to become confident with drawing abstractions from the real and concrete experiences.

Representation in multiple ways: Another important element which is required to help children move towards abstract thinking in mathematics involves helping them to develop their representational abilities. As has been stated earlier, mental representation or simply representation is the mental imagery of the object, event or process as perceived by the individual. It is now an established fact that children should be given opportunities to make their own representations of mathematical problems, processes and procedures before they are introduced to the conventional symbols. It is clear that if children are to become able and confident in solving mathematics problems, they must be able to represent mathematics to themselves and to others in language and in mathematical symbols. Many mathematics educators now believe that it is important that children express their mathematical thinking in language, through talk, before they begin to represent it on paper and before they use mathematical symbols. James (1985) reviewing the work of Bruner and others on the inter-relationship between language and thought propounds a mathematics teaching procedure which he terms 'do, talk and record'. This involves children in doing mathematics practically, and then following a five-step sequence of activities towards recording as follows:

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The learners explain their thinking to others;

They demonstrate their mental images either with objects or by sketches;

They record in writing the 'storey' of what their sketches show;

They make successive abbreviations of the process they used;

Finally, they can see the relevance of and adopt standard notations.

While forming representations of the mathematical processes and procedures are important for correct concept formation and solving mathematics problems, sharing one's representation with others in the class, helps to explore different approaches for elaborating the process or procedures and to develop multiple representations.

Developing alternative strategies: When the children can form representations, they can also develop ways to calculate and solve mathematics problems other than the prescribed ways given in the textbooks. That a child can evolve his/her own method of calculation, stems from the observations of totally non-schooled children performing calculations of various types required in their daily lives which are different from those given in textbooks. This lack of a relationship between informal and formal methods is a major cause of young children's loss of confidence with school mathematics.

Developing new strategy may not be always possible for children. But, whenever any child comes up with any new one, he/she needs to be reinforced. Searching for alternate strategy need to be a regular feature in the classroom transaction. After the discussion of any operation or procedure for solution of a problem, children may be encouraged to think of any alternate strategy of the one discussed in the class either individually or in groups.

Very often, the mathematics teacher is very rigid about the formal methods given in the mathematics textbooks and does not allow any slight deviation from those. Such an attitude does not help children to explore alternative strategies and loses interest in meaningful learning in mathematics. You, as a teacher in mathematics, need to recognize the ability of the children to build alternative strategies and encourage it as much as possible.

Problem solving and problem posing: Solving mathematics problems and the process of problem solving, although are different, have a lot of similarity in understanding the problem, suggesting and trying out different possible procedures of solution and solving the problem. Problem solving abilities can be developed when we encourage children in solving the problems independently or in groups without providing any direct support. Besides promoting problem solving abilities in children, they should be encouraged to pose problems. Posing relevant problems indicates the level of understanding of the concepts, processes and procedures of mathematics. You should encourage such practices in the classroom as much and as frequently as possible.

The processes of problem solving and problem posing have been discussed in greater detail in the unit 4 of this paper.

E7. Can problem posing help in developing alternative ways for solving mathematics problems? Justify your answer with examples.

E8. Give one example of development of number concept through manipulation of objects.

1.3.2 Mathematics Phobia

Here are a few statements of students who are very serious about their mathematics performance.

"When I look at a maths problem, my mind goes completely blank. I feel stupid, and I can't remember how to do even the simplest things."

"In maths there's always one right answer, and if you can't find it you've failed. That makes me crazy."

"Maths exams terrify me. My palms get sweaty, I breathe too fast, and often I can't even make my eyes focus on the paper. It's worse if I look around, because I'd see everybody else working, and know that I'm the only one who can't do it."

"I've never been successful in any maths class I've ever taken. I never understand what the teacher is saying, so my mind just wanders."

"I've hated maths ever since I was nine years old, when my father grounded me for a week because I couldn't learn my multiplication tables."

"When I was little, my father, who was a maths teacher, used to punish me by forcing me to do math problems."

"Maths mainly involves a lot of memorisation of facts, formulas and procedures."

"Maths is not relevant to my life."

"Maths is mainly arithmetic."

"Maths is boring."

You must have heard more of such statements of quite a large number of students who are very much worried of their performance in mathematics and have developed fear for the subject. Even some of your colleagues also might be finding the elementary school mathematics too difficult for them to understand.

From time immemorial, mathematics is considered as the most difficult of all the school subjects. What are possible reasons for it?

There are some four key features, inherent in the ways the school, mathematics has been designed to be taught which create anxiety and fear among students:

First, it is commonly devoid of any real, meaningful or supporting context. In the words of one, often quoted, famous mathematician, the trouble with mathematics is that 'it isn't about anything'.

Second, school mathematics commonly involves the use of abstract symbolism which puts the young learner in difficulty.

Third, school mathematics often requires children to use new 'paper and pencil' strategies which are not simply written versions of the mental strategies which they have already developed for themselves.

Fourth, school mathematics is often taught as a set of prescribed procedures, without helping children really understand numbers and the ways they behave. There is often more emphasis placed on 'getting the right answer' than on understanding the processes involved. And, above all, it is the precision (accuracy) that makes mathematics more difficult.

From psychological point of view, the dominant model of human learning is of the child as an information processor, attempting to derive meaning from experience by subjecting it to several modes of processing like classification or categorisation of new information and relating those to the existing experiences to derive meaning. There are three main features of the *human information processing system* which have very direct implications for introducing young children to the world of formal mathematics.

1. *Learning by induction:* As human beings we appear to be very able to engage in the process of **induction** (inferring general rules or patterns from a range of particular cases), but relatively less well-equipped for **deductive** reasoning (the opposite process of inferring particular cases from general rule). Inductive reasoning is the basic process whereby children can easily make sense of their world by classifying and categorising experience into increasingly structured conceptual structures and models. The overwhelming significance of inductive process for children's learning has long been recognised, and has long been a strong element.

2. *Limited 'working memory' capacity:* While teaching mathematics we usually are not aware that human being has a limited capacity of processing information. For example, Miller has demonstrated from a whole range of evidence that we hold only about seven separate pieces of information in our short-term or 'working' memory. This is why as adults, we can easily process in our heads a sum such as 17×9, but have much greater difficulty with 184×596. We know the procedures we must go through to get the answer to the second sum, and we can carry out each of the separate computations involved. What we cannot do is hold all the

information in our head at once. While we are working out one part, the result of the previous computation is very likely to be forgotten. This happens all the time for children with much smaller numbers and less complicated procedures.

3. *Development of 'meta-cognitive' awareness and control:* The third general feature of the human processing system which we must consider is that it is a system which not only learns but learns how to learn. When one is aware of his/her ways of thinking or learning, what the American psychologist has termed 'meta-cognition'; he/she acquires more ability to control over his/her actions and learning. We tend to engage students more in solving mathematical problems than encouraging them to elaborate the ways they can solve the problem. Solving textual problems in the prescribed ways becomes monotonous, and burdensome. But if we can explore the meta-cognitive abilities of the children, we can assign them the problems appropriate to their ability levels which they can solve through ways which they can justify.

Unaware of the information processing abilities of children, the teachers and parents drive the children to perform better in mathematics by sheer memorising and repeating the processes taught to them, predominantly deductive in nature, without developing adequate understanding and interest in the problems given in mathematics textbooks which have little relevance to their real life problems.

Besides the causes related to the information processing deficits, there are causes related to the classroom and home environment associated with the creation of fear towards mathematics learning. Some very common causes of mathematics phobia are:

Prior negative experiences with mathematics. These may be related to one or more of the following:

Unfavourable school climate: Schools where extremely rigid discipline is maintained in coverage of the course with strict adherence to the textbooks without allowing freedom to children to think and to choose alternative ways to solve mathematical problems create more tension in students towards mathematics.

Lack of encouragement from parents and/or teachers: When the child does not get any support from home and encouragement from teachers in the school in learning especially in learning mathematical concepts, anxiety to perform well in the subject increases. Added to this continuous pressure from parents and teachers for better and higher performance in mathematics develops fear for mathematics in most cases.

Lack of positive role models: Sometimes a student in higher class or a family member with exceptional mathematics abilities inspires the children in lower grades in their mathematics learning. Such a role model is rare to find and the child does not find adequate inspiration towards an abstract subject like mathematics.

Ethnic and/or gender stereotypes: There is a common feeling that girls and children from disadvantaged social groups do not perform well in mathematics. Such children are looked down in mathematics classes and are subjected to humiliating remarks.

Mathematics problems being used as punishment in school: In some schools teachers use mathematics problems as a disciplinary measure and such a practice obviously generates fear for mathematics.

The pressure of taking timed tests: The emphasis of the examinations in the school is so great that one feels as if the aim of all learning in schools is to perform well in the examinations. There is heavy pressure on children from the family and school to perform well in the tests on different occasions- weekly, fortnightly, monthly, quarterly, half –yearly, annually. Gradually, in the context of the continuous and comprehensive evaluation, the frequency of tests is more than before. Taking so many tests at regular intervals and expectations to perform better in mathematics than in any other subject are driving students to frustration.

The fear of looking or feeling "stupid" in front of others: Many students do not clarify their doubts in mathematics for the fear of being regarded as "stupid" by others. In this way they go on piling their doubts and in the long run they become handicaps in mathematics learning.

Lack of preparedness: Most of the times, with increasing number of tests they have to face, the students are not always well prepared for the class as well as for the tests. This will cause anxiety in just about any situation.

1.3.3 Making Mathematics Learning Pleasurable

There is a feeling among large sections of adults including teachers that mathematics is a serious subject and should be taught with all seriousness and there is no place of any light hearted activity in teaching and learning process. But, for young learners at the initial stage of schooling such a serious approach has invariably caused loss of interest in mathematics, developing a fear complex towards the subject and ultimately resulted in early drop out from schools.

Children can learn most of the basic concepts of mathematics being engaged in activities that give them pleasure. Every child loves to play games with other children and these games that may be the perfect medium for learning several mathematical concepts. You can take any familiar game children love to play and with slight modification you can integrate some mathematics concepts in it so that the children can acquire those

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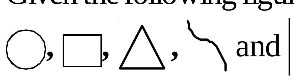
concepts while enjoying the game. In addition to these efforts, you can devise interesting activities specifically for the purpose. Here are some examples:

Number Race – Divide the students of class I into 4 or 5 groups and let them elect a leader from among them to act as the leader. Each group stands in a row facing the black board in their front. Keep a collection of pebbles in a place about 2 meters in their front. When the leader shows a numeral card, say 5, the first player in each team runs to the place where the collection of pebbles are kept and picks up 5 pebbles from it and raises his/her hand immediately after the collection. Whoever raises the hand first earns a point for his/her group after the leader ascertains the correctness of the count. The players who have completed their turn join the end of the row of their respective teams. The leader shows another card and the second player in each team races to the collection and the game continues. At the end, the team earning maximum points wins the game.

Place Value – Two teams or two players can play. Each team or player has a slate or a drawing sheet with two adjacent boxes marked **Tens** and **Ones**. Numeral cards from 0 to 9 are shuffled and put face down in a pile. The first player picks a card and decides where to put the card, either in **Tens** place or **Ones** place. The next person from the other team or the other player picks a card and puts the card in the box as he/she decides to put. Next comes the turn of the first player to pick a numeral card and this time he/she has to put it in the vacant box. The player from the second group does the same. Each player has to tell the name of the number so formed. The player or the team forming the larger number wins the round.

Given the following figures :



Using these figures, the students should be asked to draw the diagram of the objects familiar to the students. The student who draws more variety of figures within a fixed time span (say 5 to 10 minutes) wins the game.

Addition Game – It can be played between 2 or more players individually or in teams (preferably class II students). You need a pack of playing cards without the picture cards. The players sit in a semicircle. The cards are shuffled and placed face down in the center before the players. Players/teams take turns to reveal two cards, find the sum and record this as their score. Players/teams keep a running total, checking each other's calculations. When all the cards have been used, the player/team with the greatest total is the winner.

Guessing Game - This game is played between two teams (Team A and B or any other interesting names), preferably the students of higher grades forming the teams. Team A decides a number between 0 to 100 and writes it in a slip of paper and gives it to the teacher or the leader conducting the game without revealing it to the other team. Team B has to guess the number by asking questions. In a variation of this game, the team is allowed to ask definite number of questions (say 10 questions) whose answer is either 'yes' or 'no'. The score to be awarded to the team B depends on the number of questions they ask to reveal the number. If through asking one question they guess the number correctly, they are awarded 10 marks and if two questions are used to reveal the number, then 9 marks are awarded. The score goes on decreasing with increase in the number of questions. Next it would be the turn of the team B to decide the number and the team A to guess it through questioning. The game goes on in turns and after definite number of rounds, the team aggregating higher score becomes the winner.

ACTIVITY - 2

Formulate activities that will facilitate learning of measurement concepts in the primary classes.

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There are several other activities which are enjoyed by the children while learning mathematics. Creating symmetrical '**Rangolis**' on the paper using colours, introducing the children to **origami**, the art of paper folding to create various shapes of 2D and 3D objects, and familiarising with **tangrams** for producing several 2D figures are a few such activities.

Games can be a very powerful media for making mathematics learning enjoyable and at the same time meaningful for children. You can think of any game and you can easily realize that mathematical concepts can be introduced through it with imagination and enormous good effect. Let us take a very common game of 'Pithu' the children like to play in different parts of the country with different names. In this game pieces (usually 9 or 10 in number) of broken tiles or small stones, or small wooden cuboids are heaped on each other in a column and placed inside a small circle. A player has to hit it with a ball or a piece of stone to displace the pieces out of the circle. The number of pieces the player displaces is the points he scores. After that another player performs the same action and earns the points. This goes on for some rounds and at the end of it the players add the points each has earned. The player who earns highest number of points becomes the winner. You can see clearly that this game facilitates counting the objects and adding the numbers.

You can introduce more mathematical concepts in this game by introducing some variations. Here are two such variations in this game:

You can draw another concentric circle to the first one with radius nearly half a meter more than that of the first or inner circle. When a player displaces pieces from the inner circle, for one that remains outside the first circle and within the second, one point is awarded and the piece that falls outside the outer circle earns 10 points. If a player dislodges three pieces out of which one falls in between the two circles and two fall outside the outer circle, then the player earns 21 points (2×10 and 1×1). The game goes on as before. This variation helps in practicing the concept of place values and addition.

In another variation of the game, you can use pieces of different colours (pieces of 3 coloures with 3 to 4 pieces from each colour) attaching different points to coloured pieces (say 1 for each white piece, 2 for each blue piece and 3 for each red piece). The game is played as before, but the calculation of points requires multiplication and addition skills.

Besides, the activities and games, there are innumerable forms of joyful and challenging acts for facilitating mathematics learning like participating in quizzes and competitions, preparing mathematical models and charts, collecting riddles and puzzles and solving those.

ACTIVITY - 3

Select any game the children in your locality love to play. Describe how mathematics can be learnt through the game. State any two variations of this game that you can introduce. Indicate the mathematical concepts that can be learnt by making the variations.

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E9. What are the reasons for developing fear for mathematics in the classroom?

E10. State any four ways to reduce the mathematics phobia and make classroom learning of mathematics pleasurable.

For making mathematics learning pleasurable, you need to create a learner friendly environment in the classroom while teaching learning process is going on especially when mathematics is being taught. This is essential in building a free and joyful interaction between students and the teacher and among students in the classroom. Such a climate of trust and equality would help in removing anxiety and fear and make mathematics learning really pleasurable and more meaningful thus more effective.

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1.4 LET US SUM UP

Child's thinking begins with two processes: *perception* (the knowledge of objects resulting from direct contact with them) and *representation* (mental imagery of the perceived objects).

Piaget conceptualized the process of thinking as a process of organization of adaptation in a new situation reached as after equilibration of the twin processes of assimilation and accommodation.

The thinking develops in the child in four stages as conceptualized by Piaget namely (i) Sensory-motor Period (from birth to 2 years),(ii) Pre-operation Period (from 2 to 7 years), (iii) Concrete Operation Period (7 to 11 years), and (iv) Formal Operation Period (11-12 years to 14-15years).

Development of mathematical concepts in children follows the trends of cognitive development.

Pre-number concepts of matching, sorting, comparing ordering and subsidising develop during pre-school years i.e. before the age of 6 years.

While the number concepts and most of the measurement concepts develop completely during the concrete operation period i.e. before the age of 11, spatial thinking requires one or two years more to develop.

The conservation and transitivity of numbers, length, mass and weight take place before the onset of formal operation period whereas those of volume and area take longer time.

Some basic ways related to the learning of mathematics during the early stages of school learning are manipulation of objects, performing meaningful tasks in the real life situations, representation in multiple ways, evolving and using alternative strategies and problem solving and problem posing.

The anxiety and fear for mathematics are due to several factors associated with the schooling practices, disturbing the nature of human information processing, and classroom and home environments.

Mathematics learning can be pleasurable in adopting a variety of ways like performing learner friendly activities, games, preparation of models and charts, participation in quizzes and exhibitions, collecting and responding to puzzles and riddles.

1.5 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. Perception and representation.

E2. Assimilation and accommodation

E3. Matching and sorting.

E4. Concrete operation period.

E5. Formal operation period.

E6. Conservation of length is attained when the child realizes that the length of an object remains unchanged irrespective of the position of the object.

E7. Yes. Give justification.

1.6 SUGGESTED READINGS AND REFERENCES

Anghileri, Julia (ed.) (1995). *Children's mathematical thinking in primary years: Perspectives on children's learning.* London: Cassell.

Copeland, Richard W. (1979). *How children learn mathematics: Teaching implications of Piaget's research* (3rd Edn.). New York: Macmillan Publishing Co.

Dickson,Linda, Brown, Margaret, & Gibson, Olwen (1984). *Children learn mathematics.* New York: Holt,Rinehart& Winston.

1.7 UNIT-END EXERCISES

1. Describe the role of perceptions and representations in the development of pre-number concepts during the pre-school years.

2. What are the characteristics of development of mathematics concepts corresponding to those of the stages of cognitive development?

3. What are the basic ways of learning mathematical concepts and how can you make them pleasurable for children?

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UNIT 2 MATHEMATICS AND MATHEMATICS EDUCATION

Structure

2.0 INTRODUCTION

Mathematics pervades all aspects of our lives. Any person, he/she may be a farmer, daily labourer, artisan, teacher or a scientist uses the principles of mathematics in his/her day to day activities at different situations. Thus mathematics holds a key position in our life. Therefore, mathematics has enjoyed a privileged or a sheltered position in the school curriculum.

The position paper on mathematics for the National Curriculum Framework asserts that "our *vision of excellent mathematical education is based on the twin premises that* ***all students can learn mathematics and that all students need to learn mathematics.*** *It is therefore imperative that we offer mathematics education of the very high quality to all children.*" In order translate the vision; we have to critically analyze the following issues:

What should be the aims of teaching mathematics at the school level?

How our teacher can develop interest towards mathematics among the learners?

- What kinds of knowledge and skill can be developed among the learners?
- What should be the nature of mathematics learning?

In this unit, you will find answer to some of the issues mentioned above. You will definitely observe the nature of mathematics on the basis of which the mathematics education can be designed by you for your learners. Further you will realize the importance of mathematics education at elementary level.

This unit will take about *8 (eight) hours of study.*

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- State the nature of mathematics with suitable examples.
- Explain the utility of mathematics in day to day life.
- Describe what mathematical thinking is
- Describe the relationship of mathematics with other branches of knowledge

2.2 NATURE OF MATHEMATICS

As a teacher you have the experience of teaching mathematics to young children. Sometimes you might have felt that mathematics, among all the school subjects, enjoys a unique status. Do you think so?

If yes, then what are the causes?

To get the answer to the above question you have to understand the important characteristics of mathematics for which it enjoys a special place in all spheres. The nature of mathematics greatly influences the nature of teaching and learning process in mathematics. So an elementary school teacher should know about the nature of mathematics. The nature of mathematics makes the subject distinct from other subjects. Let us now discuss in detail regarding the nature of mathematics.

***Mathematics is logical*:** Mathematics is accepted as a branch of logic. According to C. G. Hempel "It can be derived from logic in the following sense:

- All concepts of mathematics i.e. arithmetic, algebra and analysis can be defined in terms of concepts of logic.
- All the theorems of mathematics can be deduced from these definitions by means of the principles of logic.

Thus, it can be said that in mathematics truth can be established with logic. The proof of mathematical statements consists of a series of logical arguments, applied to certain accepted rules, definitions and assumptions. Now observe the mathematical statement given below:

S1: *Two even numbers when added, it gives rise to another even number.*

You cannot prove this statement (S1) just by mere experienced observations. If you can take several examples and test them, then you can say that the statement could be correct. If you understand what even number is and the concept of addition, then you can prove the statement mathematically.

Any even number can be written as 2n, where n is any natural number. Now you can take two even numbers, $2n_1$ and $2n_2$ (where n_1 and n_2 are natural numbers). The sum of these two numbers is $2n_1 + 2n_2 = 2(n_1 + n_2) = 2m$, where $m = n_1 + n_2$ is a natural number. Here 2m is a number which is divisible by 2 and hence is an even number. Thus, the sum of two even numbers is an even number. This kind of logic, which uses known results, definitions and rules of inference to prove something, is called **deductive logic.**

Try to perform the following task for checking your progress:

E1. Using deductive logic prove that the sum of two odd numbers is an even number.

ACTIVITY - 1

Go through the elementary school mathematics text books used in your school. Find out ***five*** *cases where you have observed deductive logic is used.*

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Another kind of logic used in mathematics is inductive logic. Now look at the example given below:

2, 4, 6. 8, 10, 16, 36, 54, 68, and 102 are all even numbers. Now add any two of these even number and find out whether the sum is an even number or an odd number.

We find, 2 + 4 = 6, 6 is an even number.

6 + 4 = 10, 10 is an even number.

10 + 8 = 18, 18 is an even number

54 + 22 = 76, 76 is an even number and so on.

You can ask your students to add any such two even numbers and in each case they will get an even number. So studying numerous such cases we can conclude that any two even numbers when added the sum would be an even number.

This type of logic is known as **inductive logic.** In mathematics, we use logic of induction in several cases to prove mathematical results. Let us see an example from geometry.

In a plane triangle, if the measure of the 1^{st} angle is 80° and the measure of the 2^{nd} angle is 60°, then what is the measure of the 3^{rd} angle? If you will draw such a triangle with the given measures and measure the 3^{rd} angle then you will see that it will be of 40°. Similarly, you can draw a number of different types of triangles and find out the measures of the three angles of each triangle you have drawn. You will see that in each case the sum of the measures of the three angles will be 180°. If the result is true for the 1^{st} case, 2^{nd} case, 3^{rd} case, and several similar cases, then we can reasonably conclude that if ABC is any plane triangle, then the sum of its three angles would be equal to 180^0. This type of logical process of arriving at a generalized statement of relation observing the relation in several cases in similar conditions is known as **mathematical induction**. If one statement is true for *n* number of cases then it will be true for *n+1* number of cases.

Check your progress

E 2. "Every prime number has two factors."- What kind of logic is used for proving this statement?

From the above discussion we realize that mathematics is a form of pure logic. The deductive method perfected in mathematics is perhaps the strongest model of all type of logic and is a model for other logical systems. All the deductions from axioms, postulates are done by the rules of logic. Euclid's geometry is an excellent example of this and his method of breaking up the problem into 'what is given?', 'what has to be established?', and the method of establishing is nothing but a logical procedure.

Mathematics is symbolic: Let us take two statements, "Two hundred when multiplied by ten gives two thousand" or, "When the sum of any two natural numbers a and b is squared, it gives the sum of squares of a and b added with twice the product of the two numbers". But when we express it using mathematical symbols, they become

$200 \times 10 = 2000$

and $(a + b)^2 = a^2 + b^2 + 2\,ab$

You can see for yourself how use of symbols make mathematical expressions brief and clear provided you understand the notations. The symbols like those for numerals, four basic operations (i.e. +, -, $\times$, and $\div$) or figures representing line, angle, triangle, quadrilateral, circles and the likes are so familiar with everybody that not only these are easily understood but also widely used in our daily lives.

Expressing complicated and abstract ideas, the core concern of mathematics, in brief symbolic forms using common notations makes them comparatively easier to understand

and communicate to others. These systems of notations add power to mathematics and allow us to easily visualize whether a mathematical statement is correct and valid or not.

***Mathematics is precise*:** Precision is another important nature of mathematics. The term precision means 'accuracy' and 'exactness'. You can take any mathematical concept. For example, you are familiar with the concept of a cone. The definition of cone is clear and precise. 'A **cone** is a 3-dimensional geometric shape that tapers smoothly from a base (usually flat and circular) to a point called the *apex* or *vertex*'. If an object is given to you, you can definitely say whether it is a cone or not.

ACTIVITY - 1

Take a conical solid and examine as to how many curved sides and plain sides it has, and how many vertices it has.

– *Draw the picture of the cone on your notebook.*

– *Collect the objects- dice, brick, a cricket ball, conical ice cream, match box and separate those objects which are not conical in shape.*

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After doing this activity, you might have realized that, the definition of cone helps you to understand different attributes of a conical object which ultimately enables you to classify the conical and non-conical objects. The concept of cone is defined so clearly and precisely that anybody can be able to identify the conical objects in his immediate environment. So **precision** is that nature of mathematics which deals with accuracy and exactness and leaves no scope for doubt and ambiguity.

According to C. J. Keyser "The quality of mathematical thought, the certainty and correctness, its conclusion are due to the characteristics of the concepts with which it deals precision, sharpness and completeness. Such ideas admit of such precision, others do not, and the mathematician is the one who deals with those that do." As a teacher you have to focus on development of such qualities among your learners while teaching mathematics. Mathematics is characterized by its exactness and accuracy. The exactness in mathematics refers to the correctness in all aspects. Mathematics helps in developing the abilities of accurate reasoning, thinking and judgments among the children.

If you will compare other subjects with mathematics, you will see that sometimes in those subjects the answers can be written by speculating the facts or directly drawing them from experiences. Thus the subjectivity of the learner influences the answers. But there is no place of subjectivity and personal opinion or experience in mathematics. During the learning of mathematics the learners learn the values and appreciation of accuracy. He/she also learns to be accurate in approaching all the problems he/she faces in life and also precision in defining and solving the problem becomes a habit with him/her as a result of studying mathematics.

Mathematics is study of structures: The word structure means "arrangement, composition, configuration, form, order or system". Whether the mathematical concepts have certain arrangements? Have you observed any configuration in mathematical concepts? Is there any relationship between the concepts in mathematics?

If you will observe the nature of mathematics, you will see that mathematics is the study of certain structures (arrangements in the general sets). During the elementary stage, the child is going through the concept of natural numbers, whole numbers, integers, fractional numbers rational numbers and real numbers.

ACTIVITY - 2

Go through the text books and note down the definitions of different number systems with examples. Do you observe any relationship among them? Can you represent them diagrammatically?

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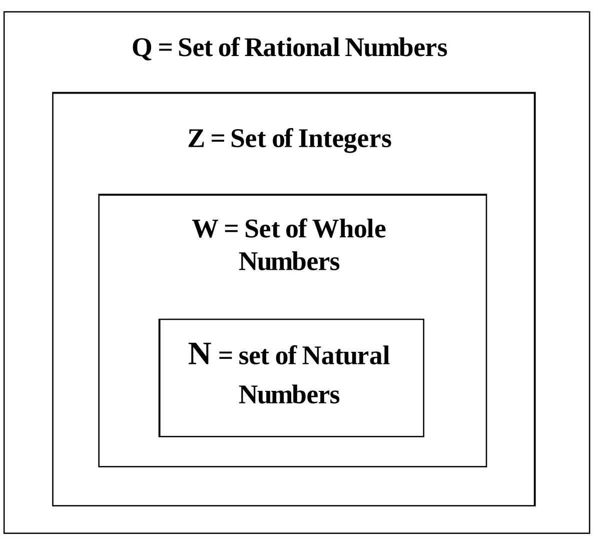

Fig. 2.1 The Hierarchy Structure of Number system

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Check your progress:

E2. "***Mathematics*** deals with precise and elegant structure". Do you agree with the statement? Give reasons for your response.

Mathematics aims at abstraction: Mahesh was teaching in grade-I. He conducted an activity in the class as follows:

He made two set of people pieces, such as male and female. Some more pieces were not used. He gave the remaining pieces to the children and asked them to place these pieces in the set. The children did the activity. Then the teacher asked them to tell why they placed these pieces as they did.

Try this activity with the students of your school, after completion of the activity now tries to answer the following questions:

Whether the children are able to place the pieces in the appropriate set?

What make them to do the task correctly?

First the children observed the properties common with all the elements in a group and on the basis of the common properties they compared the remaining pieces. If the property of the remaining piece is fitting to a particular group, then that piece is put in that group. This process is based on the principle of abstraction.

Mathematics deals with abstraction. For example, today a father's age is twice the age of his elder son. 30 years back his age was four times then the age of his elder son. What is the age of the father?

If 'x' is the age of the father today, x/2 is the age of his you is the age of his elder son today. Then 30 years back, (x-30) =4(x/2-30). Thus x=90. That is today father's age is 90 and son's age is 45.

Once noted mathematician L, Bers said "The strength of mathematics is abstraction, but the abstraction is useful only if covers a large number of special cases."

Abstraction is essential in mathematics. It is one of the amazing features of mathematics. This nature of mathematics gives rise to the development of new areas of mathematics like algebra. Algebra, a branch of mathematics deals with abstraction (one concept can be an abstraction in the sense that it is thought of as apart from material objects). Abstraction is a means of encompassing wider range of applications of mathematics.

Check Your Progress:

E3. Give an example for developing the abstract concept of triangle among grade-III children.

2.3 IMPORTANCE OF MATHEMATICS EDUCATION

Mathematics education deals with all issues at all level related to Teaching-Learning of mathematics in the socio-cultural-economic contexts and are concerned with the development and of appropriate mathematics curricula. Its importance is both theoretical (because it's mixed with nature of its link with applications of maths in day today life and interdisciplinary approach.

Keeping in view of the importance of mathematics in our daily life, the National Curriculum Framework (2005) states that clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. While there are many ways of thinking, the kind of thinking one learns in mathematics is an ability to handle abstractions and an approach to problem solving. In this section, you will learn about the importance of mathematics education for the learners.

2.3.1 Mathematics in Real Life Situation

You might have observed children playing different types of games when the school is over. The captain of a football team places the players in certain order such as 5+3+2 or 4+3+3. Similarly, if the captain of a cricket team places fielders in right position then most of the work is over. So what does the field placement require? It requires making accurate judgment of the game and of space. Games like *kho-kho*, *kabaddi* all need awareness and utilisation of space.

Check Your Progress**:**

E4. Think of a game that you played during your school days. Write down the mathematical principles involved in that game.

Let us consider the case of a farmer. The farmer normally starts planning for cultivation taking into consideration of the area of land to be cultivated, amount of seeds, fertilizers and pesticides required for cultivation, number of agriculture workers required for farming, tentative amount of money required for this. Thus, a farmer uses mathematical knowledge in his day to day activities.

ACTIVITY - 3

Observe any five people in your locality while doing any work. Write down the work they were doing. Think of the mathematical principles associated with each work. Share your views with your friends.

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Perhaps you are now convinced that mathematics spreads all the areas of your life. Similarly the children have come across different type of activity. One of the activities is described below:

The children choose a time of day convenient for several weeks and are also a time when the sun shines through the classroom window. They placed a 5cm piece of masking tape on the window sill. They placed a 30 cm piece of tape on the window sill, so that the shadow of the 5 cm tape on the window falls in the centre of the 30 cm piece on the sill.

Each day thereafter, they recorded the exact time of day that the shadow of the 5 cm piece of tape lies exactly on the 30 cm piece. They plotted the time of day on the graph. After some days the children without observing the shadow they plotted the time when the shadow exactly falls on the 30cm piece.

Here are several questions come to the mind:

How did the children select the time?

Whether the shadow falls on the 30cm piece every day at the same time?

After some days of observation how the children plotted the graph?

To answer the above questions, you should have the sense of mathematics. To answer the 1^{st} question, the concept of time measurement is required. Similarly to answer the 3^{rd} one, you need to use your experience. You have to carefully observe and record the time everyday. On the basis of the data you have to predict the time alignment for several days in advance. Here, you have to work on estimate, checks them and decide to use the data or take up another set of estimates.

From the examples we may say that mathematics is not only confined to the classroom only, rather we can see mathematics all around us. It is everything we do.

2.3.2 Mathematics and Other Branches of Knowledge

Mathematics, along with language, is considered basic and essential for human learning and civilization. There is no sphere of human knowledge which has remained unaffected by the influence of mathematics. Apart from the use of numbers, mathematical figures, formulars and processes, mathematics has influenced the ways of presentation and communication of all areas of knowledge marked by coherence and mathematical precision. In this section, we shall confine our discussion on the influence of mathematics on different branches of knowledge which are included in the elementary school curriculum. We must keep in mind that in higher levels of education, the influence of mathematics on different disciplines and subject areas is more pronounced.

Mathematics and Literature: Many consider language and literature are just the opposite in nature to that of mathematics. They feel that language is the vehicle of feelings, emotions and passion while mathematics is exact, objective and mathematical logic are devoid of any emotion and passion and have little place in literature.

But, remember Shakespeare's words, "Brevity is the soul of wit". To be brief and precise in expression is considered a sign of wit. With less number of words, if you can express more, you are more restrained and have the ability to communicate in a focused and meaningful way. That is exactly what mathematical logic is.

At the initial stage of language learning, children are given freedom to express themselves in as many words as they can use. But at each grade you need to ensure their vocabulary acquisition and it is stipulated that towards the end of primary level they should acquire around 5000 words. To have regular assessment of vocabulary acquisition, you need to use objective methods. In the upper primary grades, the children are encouraged to express within stipulated number of words i.e. they are encouraged to be precise, and comprehensive in their expression. Therefore they are trained in précis writing and paragraph writing within specific number of words and in specific length of sentences.

Again in writing poems the length or meter of lines used in the poem is also carefully chosen and meticulously adhered to throughout the poem. This helps in maintaining poetic rhythm, feeling and above all expressing the meaning. In all these, the mathematical sense prevails and controls the overall structure and sense of the literature.

Mathematics and Science: Perhaps mathematics and science share closest relationship. There is no branch of science which does not use mathematics. Take the concepts of physical science. Most of the concepts have evolved from experimentation or observation but have been established as scientific theories by use of mathematical interpretations. For example, the finding that 'water boils at 100^{0}C' is a scientific fact emerged from experiments. But, other experiments linked this to the air pressure i.e. with the increase in air pressure the boiling point of water also increases and vice versa.

To have a clear understanding of this physical phenomenon, a relationship is required involving air pressure and boiling point so that it would provide the boiling point of water at a particular atmospheric pressure. This is possible only with the help of mathematical processes. In every field of physical science, like mechanics, light, sound, chemical reactions, mathematics plays vital role in explaining these phenomena.

Ascertaining the rate of growth of various species of plants and animals, the arrangement of leaves in different plants and trees, the rate of heart beat, and measuring blood pressure etc. are a few examples of use of mathematics in biological science.

It is difficult to enlist all the areas of science where mathematical knowledge is required to understand the concepts.Infact, there is hardly any area in all branches of science where mathematics is not needed.

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ACTIVITY - 4

Take any two topics (one from physical science and one from biological science) from the science curriculum of upper primary classes and list the mathematical knowledge required for understanding the concepts included in the topics.

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Mathematics and Environment Study: In the curriculum of EVS at the primary grades, there are several topics which require quantitative descriptions and analysis based on these quantitative data. Some such examples are, planning a school campus including school garden requires the concepts of measurement of length, area to be utilized. So also in beautifying a clean classroom environment you need the mathematical concepts of symmetry along with the measurement skills. You need to use the knowledge of ratio and proportions in approximating biodegradable wastes and other types of garbage produced in the school or in its near vicinity so that provisions for appropriate waste disposal system can be developed by the students.

For a balanced and nutritious diet, the proportion of the different components of food is calculated mathematically depending on the requirement of the individual. If a child is underweight and prone to infection, then proportion of proteinous food materials along with vitamins are increased based on the calculation of the degree of deficiency of these materials. Similarly an obese (over fat) child needs less carbohydrate and fat materials in food which are calculated by the dieticians.

In studying the occupational pattern of the communities in the habitations around the school, you need to employ the knowledge of proportion, percentage, and different modes of graphical presentations to describe the pattern.

Mathematics and Geography: Geography, like science, requires the knowledge of mathematics at every step to explain and clarify the geographical concepts and phenomena. For example, in studying land forms, you need to have clear idea of measuring larger heights and variations in temperature at different heights. In calculating the temperature, humidity and rainfall in a place, drawing graphs of their indicators and in determining their interrelationships, knowledge of corresponding mathematical concepts is essential. Trigonometry helps in finding the height of a mountain.

Similarly, in determining the longitude and latitude of a place and in preparing maps, the knowledge of plane and solid co-ordinate geometry is of absolute necessity.

Mathematics and History: History is the study of events and trend of events within a specific period of time. Therefore, you should develop a sense of time and be able to

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conceive the temporal gap between important events so as to be able to effectively speculate the validity of occurrence of such events. Further, to study the relevance of past events for the present, you are required to ascertain the gap of time by drawing a time line. In all these activities which are fundamental in understanding history requires thorough understanding of measurement of instant and interval of time.

Again, drawing and understanding historical maps of different regions for different periods, and graphical presentations of socio-historical phenomena, you need to have adequate knowledge of spatial relations learnt in geometry classes.

Mathematics and Art Education: In both visual and performing arts, mathematics has crucial roles to play. In visual arts like drawing, painting and sculptures, proportion within and among the figures drawn or sculpted has to be perfectly maintained. For example, while drawing or making a human or an animal figure, the relative proportion between head, trunk, hands and feet are to be perfectly maintained otherwise the art would look disproportionate and hence lose its artistic quality. Therefore, every artist or sculptor, before actually going to paint or sculpt, prepares a sketch of the art ensuring the right proportion of the parts of the figure he/she is going to develop in the final art. If you are encouraging your students to learnt draw, paint or preparing clay models, you need to make them aware and practice to draw the sketch first ensuring correct proportion of the different parts of the figure.

In performing arts, be it singing, playing an instrument or dancing, the knowledge of beats or rhythm is the key to all such performances. For each raga one sings or plays in instrument (say flute or sitar), there is definite scale of *taal* associated with it. Changing the slightest beat in the scale may disturb the tune and may lose its musical value. The music scale, in written form using musical notations, looks like a pictorial graph with notations arranged in a specific order and specific distances among the notes indicating the definite time gap needed between producing the consecutive notes. At the beginning stage, the learners are habituated with the variations in tones and rhythms through counting the beats. Similar rhythm is maintained in different dance forms and styles through performing according *taal* produced by oral counting and then by the beating of drums or some percussion instruments. One needs to differentiate the tonal variations and the differences in *taals* through the rhythmic variations in counting. The perfect relationship between the nature of performing arts and the mathematical principles associated with each has to be clearly understood by everyone desirous of becoming a successful performer.

Mathematics and Physical Education: Wherever you desire to maintain some form of order, you need numbers and physical education provide the brightest example of it. Whether it is mass drill, or individual yoga or performing aerobic exercises, you find wide use of numbers. Recording exact time for each event while performing in sports, building strategies for different games and athletics need mathematical expertise which is now being included in the training of the coaches and trainers.

2.3.3 Mathematics and Problem Solving

According to N. J. Fine, 'a problem that is posed represents an outpost to be taken, a simple engagement in our conquest of the unknown.' The Hilbert's Thesis on Mathematics quotes, 'mathematics is forward looking, being more connected with solving outstanding problems and creating powerful new concepts and methods than with merely contemplating knowledge that has already been won'.

Thus, learning mathematics is and developing problem solving abilities among the learners are nearly synonymous. Mathematical problem solving is used in most of the literatures, is far more than solving the word problems. George Polya, in his book *Mathematical Discovery*, defines problem solving as the conscious search for some action appropriate to attain some clearly conceived, but not immediately attainable aim.

The problem solving ability of the learner is dependant on the acquisition of mathematical knowledge. Problem solving is far more useful as a guide to preparing children to be effective problem solvers in every day life. Let us consider an example:

A typical verbal problem is as follows:

Mohan sold 8 boxes of candy and Gouri sold 3. How many boxes must Gouri sell in order to sell as many as Mohan?

Such type of problem can be solved by using mathematical knowledge. If the child is well acquainted with the language used in this problem and corresponding mathematical operation, then he can solve this particular problem as well as similar types of other problems. Let us discuss how to develop the problem solving processes.

If appropriate experiences are provided to the learner, then young children can learn basic problem solving processes.

First, the problem should have few abstract concepts and the child must be able to make connection among the data given in the problem.

Second, the problem requires several steps to arrive at a solution. This causes the children to organize, reflect and record intermediate solutions.

Third, the ideal problem has multiple answers rather than one correct answer. The children should be allowed to search those multiple answers.

Fourth, problem requires analysis and synthesis of information. Complex problems help the children to solve life related problems.

The processes like observing, inferring, comparing, copying patterns with objects, using trial and error, classification of objects and data and using appropriate strategies are involved in problem solving. These processes can be well developed through learning of mathematics. Thus, mathematics learning is important to not only solve the mathematical problems but also the problems we confront in our day to day situations.

2.3.4 Ability to Think Mathematically

To understand the learner's ability to think mathematically, let us observe a problem, for example, to find out the relationship between HCF and LCM of any two natural numbers.

The problem here is to find out the relationship between the H.C.F. and L.C.M. of any two natural numbers. How are you going to solve it? Are you going to focus on a few pairs of natural numbers and study the nature of HCF and LCM of such pairs of numbers? If you are doing so, then you are ***specializing***.

Number Pair	HCF	LCM	Remarks
(4,6)	2	12	
(3,8)	1	24	
(6,6)	6	6	
(3,7)	1	21	

Do you observing any pattern? Does it make you ***conjecture*** (or making a reasonable guess) a rule? What is the ***general*** rule?

Is the LCM is always greater than HCF? Is the HCF of any two numbers is less or equal to each number in the pair?

You need to check if your generalization is right. This means that you need to ***prove your conjecture*** (guess). You have to start from certain assumption, and arrive at the result by a series of steps.

By observing the chart, you may arrive at the general rule as follows:

> The HCF of any two numbers is either less than both the numbers or equal with them, but it should not greater than each of the numbers.
>
> The HCF of two prime numbers is always 1 and the LCM of them is the product of the numbers.
>
> The product of the two numbers is equal to the product of the HCF and LCM.

Can you derive any other general rules from the chart?

Now you may be curious to know whether these general rules are applicable to three different numbers. Are these rules applicable to numbers more than 10,000? In this case, you are ***posing a problem.*** Once you pose a problem you will definitely test your conjecture, and prove it. In case it cannot be proved, you may go back and make modifications in your conjecture and generalizations and try to prove it again or otherwise reject it. So during the process of problem posing and problem solving the mathematical thinking goes in this line:

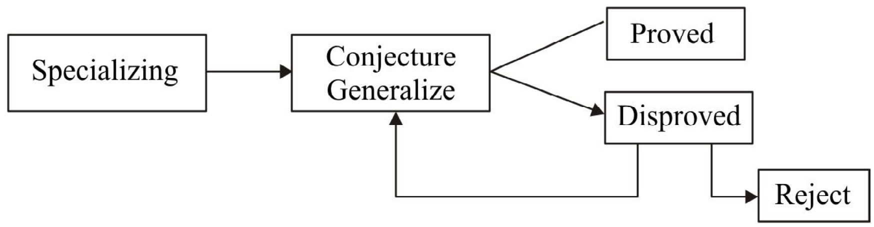

Fig. 2.2 Thinking Mathematically

To define mathematical thinking, the renowned mathematician H. Weyl stated "By the mathematical thinking, I mean first that form of reasoning through which mathematics penetrates into the sciences of the external world and even into our every day thoughts about human affairs."

Thus, mental abilities like ***thinking precisely, articulating clearly, think logically and systematically and generalizing from patterns*** helps us immensely in our real life situations. Those are the process of mathematical thinking.

Learning mathematics serves both as a means and an end. It is a means to develop logical and quantitative thinking abilities. At the early grades, children's learning of mathematics should be a natural outgrowth from children themselves. Such experiences must be interesting and should challenge their imagination, so that while observing any natural phenomena they can think mathematically.

Intuitive thinking and reflective thinking develop mathematical thinking among the learners. Intuitive thinking means learning by experimenting with concrete materials, through experiencing ideas in various concrete ways, and by visualizing ideas relying on analytical thought processes. But reflective thinking comes later. Reflecting thinking means being able to reason with ideas without needing concrete material. The processes of reflective thinking include reflecting, inventing, imagining, and playing (dealing) with ideas, problem solving, theory building and generalizing.

Check Your Progress:

E5. Give examples from intuitive thinking and reflective thinking. How they help in thinking mathematically?

2.4 LET US SUM UP

The proof of mathematical statements consists of a series of logical arguments, applied to certain accepted rules.

Precision is that nature of mathematics which deals with accuracy and leaves no scope for doubt and ambiguity. Give a precise statement, it is either true or false.

The mathematical structures are elegant and precise. Mathematics enables the learners to make precise statement. The habit of expressing oneself clearly and accurately can be cultivated by learning mathematics.

Correctness in the procedure to solve any mathematical problem, correctness in the method and result is one of the natures of mathematics.

Mathematics learning aims at abstraction. Abstracting is the ability to look at the set of objects thus classified and identify the scheme by which the classification was accomplished.

Every human activity involves some mathematical principles and mathematics is useful in all spheres of life.

To verify a mathematical statement, you need to prove it for all cases. If it is not true for even a single case, then it is not true at all.

Mathematical thinking consists of solving problem and posing problems. Mathematical thinking requires the skills of precise thinking and logical reasoning.

Mathematical knowledge enables us to solve daily life problems. The processes like observing, inferring, comparing, copying patterns with objects, using trial and error, classification of objects and data and using appropriate strategies can be developed through learning of mathematics.

2.5 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. Deductive logic

E2. Yes, the habit of clarity, brevity, accuracy and certainty precision in a written or spoken expression are formed and strengthened by the study of mathematics. Mathematical concepts and symbolism (use of symbols) provide a means of concise expression which is elegant in its simplicity and exactness.

E3. Give the child a large number of triangles made of drawing sheet. Ask the child to explain each model. The child may be helped to derive the common characteristics of a triangle on the basis of its shape, number of sides, number of angles and number of vertices. Then withdraw the models from the child. Ask him to draw a picture of a triangle and describe it.

E4. The answer can be written as per the example given in the text.

E5. Suppose the child was asked to add 18 and 17. The child took18 numbers of sticks and make them a bundle of 10 and eight single pieces. Then he took 17

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sticks and converted those into a bundle of 10 and seven sticks remains as single pieces. He got two bundles and 18 no. of single sticks. Again the child handles the 15 numbers of single sticks and makes them a bundle of 10 sticks. So all total he had 3 bundles and 5 single sticks give 35 as the result. The thinking involved in this process is intuitive thinking. On the other side, when the child adds the two numbers without taking the help on concrete material but with an algorithm then he uses the reflective thinking process. At the initial stage of learning a particular concept intuitive thinking is very useful. But, when the child gets acquainted with the process, then the process of reflective thinking starts.

2.6 SUGGESTED READINGS AND REFERENCES

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2.7 UNIT END EXERCISES

1. ‘Mathematics is logical’ - how will you use this nature of mathematics during teaching of mathematics at elementary grades (?)

2. Mathematics is the study of structures, how?

3. Mathematics education is important for developing mathematical thinking skills- give examples.

UNIT 3 GOALS AND VISION OF MATHEMATICS EDUCATION

Structure

3.0 INTRODUCTION

Mathematics occupies an important place in all civilizations, past and present, throughout the world. As has been discussed in earlier units, Mathematics pervades all branches of knowledge and all walks of life. The development and innovations in science and technology, which are the main driving force of unprecedented change happening across the world, are based on the application of mathematics. The whole world seems to be mathematically designed so much so that Sir James Jeans, the famous British astronomer, once told, "God is a supreme mathematician who created this well ordered and systematic Universe".

In spite of its pervasive use, mathematics is perceived as a difficult subject to master because of its abstractness. Most of us believe that mathematics is a difficult subject, beyond the understanding of common man. Most of the pupils as well as quite a significant number of teachers are literally afraid of mathematics and it is no wonder that they develop mathematics phobia. You have already learnt about mathematics anxiety and mathematics phobia discussed in the first unit of this course. Some teachers who have difficulties in understanding mathematical concepts and processes pass on their confusion to their students.

There are a quite a large number of questions that come to our mind particularly when we are teaching mathematics to young children- 'why should one learn a difficult subject like mathematics?', 'How a highly abstract subject like mathematics with its symbols, operations and logics can help in our daily life?'. 'What are the immediate returns of learning mathematics?'. 'Can mathematics teaching and learning be a fun?' Answer to these and such other questions would help us develop a clear vision of mathematics learning and help us to demystify our awe and phobia of mathematics.

In this unit we have tried to develop a proper vision of mathematics through the discussion of the broader and specific aims and objectives of teaching and learning mathematics at the elementary level of schooling.

For completing this unit, you shall require about *07 (Seven) hours of study*.

3.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- State broader and narrower aims of teaching and learning mathematics education
- Identify the indicators of effective mathematics education.
- Create conducive learning environment in the school for Mathematization.

3.2 AIMS OF MATHEMATICS EDUCATION

Mathematics education has its own aims. According to David Wheeler, it is "more useful to know how to Mathematize than to know a lot of mathematics". It is echoed in the National Curriculum Framework 2005, "Developing children's abilities for mathematization is the main goal of mathematics education." According to George Polya, there are two kinds of aims in mathematics for school education such as broader and narrower aims.

3.2.1 Broader and Narrower Aims

Before going to discuss the broader and narrower aims of mathematics education, let's do the following activities:

ACTIVITY - 1

Write down the areas and works where mathematics holds a key position.

....

....

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ACTIVITY - 2

Why do you think that children should learn Mathematics?

....

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....

As stated above, development of abilities for mathematisation is the ultimate goal of mathematics education. But, what is 'mathematisation'? Literal meaning of the verb '*mathematize*' is '*to reduce to or as if to mathematical formulas.*' In general, the term "mathematization" refers to the application of concepts, procedures and methods developed in mathematics to the objects of other disciplines or at least of other fields of knowledge. One is said to have acquired the ability to mathematize when he/she is able to demonstrate orderly and systematic ways of expression and behaviour with mathematical precision in all his/her activities. Mathematics education from the earliest level should aim at such higher aims besides mastering the usually practiced computations and constructions.

Developing the ability of mathematization which is regarded as constituting the higher aims of mathematics, includes developing such abilities as problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualization, representation, reasoning and proof, making connections, mathematical communication including developing aesthetic feeling. Such a higher aim in mathematics "is to develop the child's inner resources, to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems" (NCERT, p.46),

Problem solving: Problem solving is an important life skill which suggests a shift from memorization to understanding of concepts and ability to apply those in both familiar and unfamiliar situations, whether in daily life problems or in problems given in the textbooks. The problem solving skills include skills of observation, experimentation, estimation, reasoning and verification. Abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guesses and verification are useful in many problem contexts. How can you measure the length of a house? Students may utilise their experience of measuring a table and can use fingers, foot, hand, stick, rope, scale, measuring tape etc. After some trials students may use measuring tape when they are asked to measure a long space. When children learn a variety of approaches their toolkit gets richer and they also learn which approach is best for the problem they are facing.

The emphasis in this approach is more on the process involved and not just on the product. Good problem solving functions on the assumption that real life problems are open ended and have more than one solution and hence require the use of reasoning, analysis etc. for taking appropriate decisions. Problem solving activities also help to connect mathematics to the real world.

Use of heuristics: It is generally believed that mathematics is considered to be 'exact' where one uses 'the appropriate formula'. But one can use alternative processes and interactive methods to solve a problem. As we have already seen that a problem can be solved in more than one way. When one solves a problem in a way different from the one given in the textbooks thought to be the only way, he/she feels a sense of discovering the alternative. This encourages the learner to try different hunches for solving the problems. One who uses such heuristics becomes, in the long run efficient in solving real life problems. Most scientists, engineers and mathematicians use a big bag of heuristics- a fact carefully hidden by our school textbooks.

Estimation and approximation: Estimating quantities and approximate solutions when exact ones are not available are considered essential skills required for scientific investigations. When we estimate the total expenditure in organising a cultural function, or approximate time for completing a task, we may not get the right answer but surely gain advantage of reaching nearer to the solution. In many cases students use this skill to employ these approximations in solving more complex problems. School mathematics, therefore, can play a significant role in developing and refining such useful skills which is not found in the textbooks and in our classroom transactions.

Optimization: Optimization means utilization of available conditions and resources to the fullest extent which is never included in the school mathematics curriculum. The skill of optimization helps to examine whether the conditions provided for the solution of a problem are sufficient and whether all the conditions provided can be utilized in solving the problem. Let us consider two simple problems in Arithmetic:

1. Ajay's annual income is Rs.3.5lakhs. He wants to purchase a house worth Rs. 15 lakhs. After how many years he can purchase it if he does not like to incur any loan for it?

2. Milli wanted to purchase small gifts for five young cousins(say A, B, C, D and E) in her relations and she had Rs.100 with her. Each child shall get Rs. 5 more than the immediate younger cousin. How should she distribute so that the amount is fully utilized (i.e there would be neither surplus nor any deficit)?

In the first problem, there are several conditions wanting to find out a solution. The two given conditions (the annual income and the total cost of the house) are not sufficient for determining the time period for the purchase, without knowing the exact annual savings and other impending expenditures or escalation and depreciation in the value of the house over the years.

The second problem presents a situation where optimization of available funds is possible under the given conditions.

Considerations for optimization may not be always easy, but intelligent choice based on best use of available information is a mathematical skill that can be taught even at primary school stage.

Through several problems in all sections of school mathematics, like proofs of geometric deductions, constructions of geometric figures, algebraic equations or identities, or solving any arithmetic problem, skills of optimization can be developed which have immense relevance for solving the real life problems.

Use of patterns: Study of patterns requires students to recognize, describe, and generalize patterns to arrive at rules and formulae. If children are made to identify regularities in events, shapes, designs, sets of numbers they will realize that regularity is the essence of mathematics. It provides the basis for inductive learning too. Exploring patterns is both fascinating and interesting and can be made also a fun activity for children.

Representation: Modeling situations using quantities, shapes and forms is the best use of mathematics. Such representations aid visualization, clarify essentials, help us discard irrelevant information. Again, what we need are illustrations that show a multiplicity of representations so that the relative advantages can be understood. For example a fraction can be well understood through an object and its cut pieces but can also be visualized as a point on the number line. Both representations are useful and appropriate in different context. Learning this about fractions is far more useful than arithmetic of fractions.

Reasoning & proof: Mathematics is based on reasoning and proof. Two persons may have same answer to a particular question, in different ways. This can be observed in the following example:

What is the next to 3, 15, 35, 63, 99, …. .?

A, provided the result through the following pattern $2^2 – 1$, $4^2 – 1$, $6^2 – 1$, $8^2 – 1$, $10^2 – 1$, $12^2 – 1 = 143$

B presented it as 3, 3 + 12, 15 + 12 + 8, 35 + 12 + 8 + 8, 63 + 12 + 8 + 8 + 8, 99 + 12 + 8 + 8 + 8 + 8 = 143.

The process of reasoning and proof is important in mathematics. So school mathematics should encourage proof as a systematic way of argumentation. The aim should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning.

Making connection: Mathematics has been making connection within mathematics and between mathematics and other subjects of study. Children learn to draw graphs in mathematics class, but fail to think of drawing such a graphs in their project work , or in solving problems in physics and in other subject areas. Mathematical symbols and logic have wide implications in solving problems in science and presenting facts effectively. The skill of connecting mathematical knowledge with other areas of curriculum and with the problems of real life needs to be initiated from very early stage of mathematics learning.

Mathematical communication: Precise expression and unambiguous use of language are important characteristics of mathematics education. Using mathematical symbols, language, operations etc. makes mathematics more meaningful and systematic. X is two times and 52 more than Y, and if Y is 75 what is X? Can precisely express as X = 2Y + 52 = 2x75 + 52 = 202. It helps the persons to communicate their experience and views in a precise manner.

These broader aims of learning mathematics have been grossly neglected in our school curricular and co-curricular activities. We mostly focus on acquisition on some fundamental content areas. NCF-2005 is explicit in stating that "the narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy- numbers, number operations, measurements, decimals and percentages" (p.42). While the acquisition of basic content knowledge is necessary, learning content for content sake only encourages rote learning without developing proper understanding and skill in using to achieve the broader aims of mathematics education. The curriculum as well as classroom transactions need to synchronize the two aspects.

Considering the broader and narrower aims of mathematics education, the followings are some of the major aims of mathematics education:

To develop the powers of thinking and reasoning.

To solve mathematical problems of daily life.

To understand and acquainted with the environment and culture

To prepare the child for various technical and general future professions.

To prepare the child for higher study.

To develop in the child the power for invention.

E-1 State any five aims for teaching and learning mathematics concept.

3.2.2 Specific Aims

The specific aims of mathematics education helps to design suitable methods for planning effective classroom learning process, curriculum, guide to prepare TLMs, prepare evaluation procedures etc. Thus it is desirable to write the specific aims in action verbs, pin pointed, short, achievable etc. The following are some of the specific aims of mathematics education:

To ensure a good start to the students in learning mathematics.

To give clarity on fundamental concepts and processes of the subject.

To create love, faith and interest for learning mathematics.

To develop in them a taste and confidence in mathematics.

To develop appreciation for accuracy.

To acquaint them with the relation of mathematics with their present as well as future life.

To see aesthetics in mathematics.

To develop in them the habits like regularity, practice, patience, self reliance and hard work.

To apply mathematics in other subjects.

To acquaint them with mathematical language and symbolism.

To prepare them for the learning of mathematics of higher classes.

To prepare them for mathematical exhibitions.

Instructional objectives for teaching of Even and Odd numbers?

The students will able:

To *divide* a collection of *objects* into two equal parts,

To *identify* even and odd numbers of three digit numbers,

To *differentiate* between even and odd numbers,

To give *example* of even and odd numbers,

To give *examples* of real *life situations* where even and odd numbers are used.

E-2 State any two reasons for stating specific aims for teaching and learning mathematics concept.

E-3 Which of the following are instructional objectives of mathematics?

i) To develop in the child the power of thinking and reasoning.

ii) To develop in child a scientific and realistic attitude towards life.

iii) To apply addition of two digits numbers in solving problems of daily life.

iv) To understand and acquainted with different coins.

v) To perform computations with speed and accuracy.

vi) To use appropriate symbols for "Fifteen is five more than ten".

vii) To make reasonably good approximations and estimate measurements.

viii) To apply formula of direct variable to solve simple problems of day to day life.

ix) To recognize order and pattern.

x) To indentify even and odd numbers of three digit numbers.

ACTIVITY -3

Write the instructional objectives for teaching learning:-

i) Place value of three digit numbers

ii) Simple interest

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3.3 VISIONS FOR SCHOOL MATHEMATICS

As teachers in mathematics, we are to facilitate children to develop love for learning mathematics. But our activities in the classroom, our teaching style and our interactions with children depend on our beliefs and expectations about mathematics education. How do we view mathematics? 'Do we view it as an abstract, dry and complex subject?' Is rote memorization the only method of learning mathematical concepts?' 'Is mathematics education only meant for a few capable children? Or 'do we view it as an interesting and meaningful subject which can be learnt by everyone?' We can ask ourselves such other questions. In short, what is our vision of mathematics education?

Visioning is associated with the ideal that we aspire to achieve. A higher and lofty vision makes us to strive for higher quality of education. This is equally applicable for mathematics education. Let us see what has been the National vision of mathematics education at the school stage as reflected in different National reports and documents.

National Policy on Education, 1968, which shared the dream of all round development in every field through technological and scientific advancements of the National Commission on Education 1964-66,remarked that "advent of automation and cybernetics in this century marks the beginning of the new scientific industrial revolution and makes it all the more imperative to devote special attention to the study of mathematics".

The National Policy on Education 1986 proposed that “Mathematics should be visualized as the vehicle to train a child, to think, reason, analyze and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning”. The shift in treating mathematics education as an instrument for the National development to the development of child’s abilities is perceptible. This has been carried forward further in the National Curricular Framework 2005 wherein it is envisaged child’s active engagement in mathematics learning involving “enquiry, exploration, questioning, debates, application and reflection leading to theory building and creation ideas/positions in mathematics”.

Visions for School Mathematics: Keeping the above stated higher objectives of mathematics education in view, the NCF 2005 have spelt out the vision for school mathematics :

Children learn to enjoy mathematics: This is important goal, based on the premise that mathematics can be both used and enjoyed life-long, and school is best place to create such a taste for mathematics. On the other hand, creating (or not removing) a fear of mathematics can deprive children of an important faculty for life.

Children learn important mathematics: Equating mathematics with formulas and mechanical procedures does great harm. Understanding when and how a mathematical technique is to be used is always more important than recalling the technique from memory (which may easily be done using books or browsing appropriate websites), and the school needs to create such understanding.

Children see mathematics as something to talk about, to communicate, to discuss among them-selves, to work together on. Making mathematics a part of children’s life experience is the best mathematics education possible.

Children pose and solve meaningful problems: In school, mathematics is the domain which formally addresses problem solving as a skill. Considering that this is an ability of use in all of one’s life, techniques and approaches learnt in school have great value. Not only confining to development of problem solving skills, mathematics also provides an opportunity to make up interesting problems, and create new dialogues thereby. Posing problems reflects the levels and quality of learning besides developing the ability of creative inquiry in the children.

Children use abstractions to perceive relation-ships, to see structures, to reason about things, to argue the truth or falsity of statements. *Logical thinking* is a great gift of mathematics can offer us, inculcation, such habits of thought and communication in children is a principal goal of teaching mathematics.

Children understand the *basic structure of mathematics*: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school mathematics, all offer a methodology for abstraction, structuration and generalization. Appreciating the scope and power of mathematics refines our instincts in a unique manner.

Teachers engage *every child* in class with conviction that everyone can learn mathematics. Settling for anything less can only act towards systematic exclusion, in the long run. Adequately challenging the talented even while ensuring the participation of all children is a challenge, and offering teachers means and resources to do this is essential for the health of the system.

3.3.1 Children and Mathematics Education

Imagine that we are in an elementary school where a teacher teaching mathematics in a class. What we are going to see? We may see that a teacher is standing at the black board, chalk in hand, explaining something, writing solution on black board, asking students to copy the answers from black board/books, practice the exercise in the text book, sometimes students were answering teacher's questions or repeating the teacher's words, teacher is active and students are passive listener.

Problems in teaching and learning of mathematics:

***Fear and Failure*: ** Most of the students, peers, teachers, parents etc. have given priority to teaching and learning mathematics at the elementary level although most of them thought that it is a difficult subject. Lack of awareness of objectives is also another cause of fear and failure. Failure to recognize place value leads to failure in four operations in mathematics.

***Disappointing Curriculum*:** Unattractive and Loaded mathematics curriculum created disappointment among the students. Most of the mathematics curriculum emphasizes on procedure, formulas, mathematical facts and memorization of concepts. The textbook and syllabi on mathematics are rigidly prescribed. The mathematics curriculum is far away from the real life.

Inadequate Learning Materials: For majority of children in elementary schools textbook in mathematics is the only resource material available to them. Further, most of the textbooks, mathematics textbooks as well, are mostly content loaded and prescriptive. The student find very little scope for pleasure and fun in learning mathematics from the texrbooks. There is hardly any other materials available to the children especially those who are in rural and remote areas.

***Crude Assessment*:** Most of our mathematics curriculum emphasized on memorization of formulas. Our class room teaching process is also examination oriented. In our school, different tests are designed to assess student's knowledge of procedure and memory of formulas and facts. Questions are set not to expose student's experience but to get a fixed answer. For example students free to answer — + — = 8 than 2 + 6 = —? More over similar types of assessment procedure are applied in formative assessment as well as summative assessment. This type of crude methods of assessment encourages perception of mathematics as mechanical computation.

***Inadequate teacher preparation*:** Teaching and learning of mathematics in elementary level is purely depends on preparation of teachers, her own understanding, preparation of teacher on pedagogic techniques and student's preparedness. As there is acute dearth of mathematics teachers, it forced other teachers to teach mathematics in the classes in compulsion. They are mostly depending on the text books. Most of the teachers also assume that they know all the mathematics required for the elementary level. So there is a lack of teacher preparation in teaching of mathematics.

Teaching learning process: Teaching learning process in mathematics at the elementary level are not attractive because i) Bookish knowledge in the class creates dissatisfaction, ii) School mathematics learning becomes charmless, dull, uninteresting, and stereotype, iii) Emphasis on rote learning, iv) Emphasis on teaching, not on learning, v) Development of Understanding, Application and skill are ignored

Lack of interest: Most of the school children find learning of mathematics difficult and lose their confidence in mathematics. The teaching learning process in mathematics is not joyful and attractive. Even the students don't know what benefit they will get after learning mathematics. So students lack their interest and attitude towards mathematics.

3.3.2 Mathematics Education Beyond Classroom

In our elementary schools, the textbooks are considered to be the only sources of learning. In most of the cases, the children are discouraged to refer any other books besides the textbooks. Because the examination question papers are set on the basis of the prescribed textbook. Some questions arise to the mind: Is the textbook the be-all and end-all of the all knowledge? Are the writers having sufficient knowledge about child psychology? Is child's environment taken into consideration? We do not have satisfactory answers to theses and such other questions but we accept the textbook as the prime source of teaching learning process.

Children learn not only from the teacher, but also from interaction with other children, environment around them. Children learn through their senses such as, smell, touch, taste, hearing and vision. Activities involving more than one sense will help children learn better. Children learn easily if the teaching learning process is interesting, activity based, allows for active participation and thinking at their level, joyful and relevant to the child's immediate environment. More over children learn at all time, every- where, in class and out- side the class i.e. home, play ground, market etc. So our aim is to make learning in school a happy experience breaking the boundary between learning within classrooms and learning outside the classroom.

Look at the following situations:

Situation 1: *There are seven toys in a basket. Three of the toys are broken. Children are asked to tell the number of toys in the basket.*

Ranbir : There are seven toys in the basket.

Yash : There are four toys in the basket.

Ratana : There are ten toys in the basket.

Ashish : I cannot say as the number may vary from person to person.

Soumya : The question needs some corrections.

Whose statement is right? And why?

Explanation of the statements:

Ranbir : The toys are broken but all the broken toys are in the basket .

Yash : 7 – 3 = 4

Ratana : Three toys are broken, so there are six broken toys. So, 7-3+6=10

Ashish : I cannot say as the number may vary from person to person as a toy can broken into 2/3/4………pieces.

Soumya : The question needs some corrections i.e. Now how many good toys are there in the basket? Then the answer will be 7-3=4.

After going through the "Explanation of the statements" check your opinion about the statements whether it is right/wrong? One may give full mark to Yash, as it the most common right statement. But, think of the others statements.

Look at the following questions:

What is the main objective of this question?

Has Yash achieved the objective?

Have Ratana , Ayush, Ashish and Soumya achieved the objective?

Yash's statement is just recall of the answer in the class room. Ranbir, Ayush, Ashish and Soumya look the question in a different way. They have taken their experience, life situations, thinking etc. into consideration. They have gone beyond the class room situations.

Situation 2: *The teacher asked the students to solve the problem* $18 \times 12 = ?$ *without using writing materials.*

Papali : $18 \times 10 + 18 \times 2 = 216$

Jiban : $12 \times 10 + 12 \times 8 = 216$

Rahul : $18 \times 12 = 216$

Akash : $20 \times 12 - 2 \times 12 = 216$

Analyze the answer of the above four students and explain who have connected knowledge to life outside the school, and how? If we allow children to solve mathematical problems by their own methods, we would find and amazing variety of thought processes.

Scope of connecting knowledge to life outside the school:

Market**:** Most of the students might have gone to a market with their parents. They must have observed and participated in the way of buying and selling of goods and also the approach of the buyer and seller. You should take the advantage of real situations and utilized the experience of the students in calculation of profit & loss, preparation of bills, process of weighting, counting of money, amount and price etc.

Garden: Students prepare plots in home, schools and also in playing with peers. During that time they may not know counting, measurement, construction of angles, different types of geometrical figures, areas, different lines, average etc. but they may do it using their perception. How can one prepare a plot of two meters each side? You may share the experience of the students in these acivities and it will be amazing to find out that they have already acquired a lot of mathematical concepts which requires slight refinement for acquiring formal knowledge and understanding of the concepts.

Real life: A frog climbs 30 meters on a pole in a day and slides back 20 meters in a night. If the pole is 70 meters high, then how many days the frog will take to climb to the top of the pole? Most of our students of upper primary classes may calculate the answer as 7. One student told the answer is 5 as the frog climbs 40 meters in 4 days and in the fifth day it reached on the top i.e. 70 meters. Students get opportunities to work in a natural setting, they work according to their own perceptions. So their real life experience must take into consideration.

Making designs**:** Students covered their note books, paint pictures, decorate their houses, plants trees in garden, design their playing kits etc. At that time, are they using mathematics? How many match sticks are required to design your name? Teacher must observe the process of making design and utilized it in class room.

Festival: We celebrate many festivals in our homes as well as in our schools. Students heartily involved on the Independence day, Republic day, Teacher's day, Children's day, Saraswatipuja, Ganeshpuja, Id, Christmas etc. They involve themselves in different activities to make these special occasions memorable. They go to market to buy various materials, decorate the school, distribute sweets, calculate expenditure etc. at that time they also learn mathematics.

***Play ground*:** Students are playing Kabaddi, football, cricket, volley ball, basket ball and also many indoor games. They frame their rules of their own. Prepare play ground in a group. Students construct Circles, Rectangles, Squares, triangles etc.In their play ground without knowing the rules of construction. They count individual and group scores through their own strategies. Ramesh scored two fours, two twos and a single in a cricket match. How can he calculate his total score without knowing multiplication?

E4. How can you go beyond the text book for teaching and learning mathematics concepts.

3.3.3 Making Mathematics Learning Joyful

The mathematics learning in school is very often dull, un- interesting, difficult and boring. One of the most important reasons is "joyless experience" of the teachers and students. The Learning Without Burden Report rightly said "joyless learning a situation where a lot is taught but little is learnt or understood". So, while planning for mathematics learning it is important to know, what students really enjoy to do while learning mathematics. What are their needs and interests? How students actually learn? etc.

ACTIVITY - 5

List the areas of interest of your students and identify which of these areas are suitable for learning mathematics.

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How mathematics learning can be joyful?

- Each child's learning experience must be taken into consideration.
- Establish linkage between concrete objects and abstract concepts in mathematics learning.
- Use Mathematical games, puzzles and stories to create curiosity among the students.
- Developing Mathematics magic makes the mathematics learning interesting.
- Use diverse materials like flash cards, stones, sticks, objects, pictures, cutouts, charts, calendars, playing cards, cartons etc.

Organize mathematical quiz, debate, seminar etc.

Collections of photographs of great mathematicians.

Ensure opportunities for learning by discovery.

Reduce wide gaps between mathematical theory and practice.

Linking mathematics with life situation.

Asked the students to collect local games, songs, dramas etc. and convert it to mathematics learning.

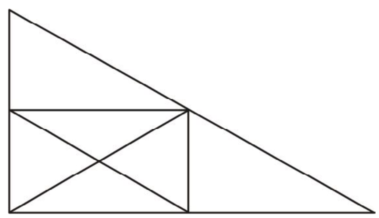
Take students to outside the classroom to observe the nature.

Allow the child to take independent decision .

Children will learn more mathematics if the mathematics learning is joyful.

Do not impose adult's views on the students as it restricts the child's creative expression.

Example: How many right angles are there in the picture?



Example: Use an identical digit for three times to make 30.

E-5 Give an example of joyful mathematics learning.

3.3.4 Creating Conducive Learning Environment For Mathematization

Most of the elementary school students do not show their enthusiasm for learning when mathematics is taught in the class. A majority of children have a sense of fear and failure regarding mathematics, so they give up difficult mathematical problems. Most of the teachers are guided by their own level of understanding and not of their pupils. Teachers also lack confidence in the teaching learning process. In most of the elementary schools the environment is not fit for teaching and learning of mathematics.

ACTIVITY - 6

Is environment of your school/ school of your village suitable for learning mathematics? Explain.

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Suggestions for learner friendly environment :

***Knowing the children*: ** Knowing the children not only important in mathematics learning but also important in total education system. In learning of mathematics teacher should i) Know every individual child in the class, ii) Praise children when they attempt to the mathematical problems, iii) do not except children to do tasks which we ourselves do not do, iv) know merits and demerits of every child in solving the problems in mathematics, v) give sufficient time to solve the mathematical problems etc.

***Teaching learning programmes*:** Most of the students thought, mathematics is a borrowing subject. Teacher should adopt interesting teaching learning programmes , which will create positive attitude among the students towards learning mathematics. The mathematics teacher should i) greet the children every day with a mathematical joke, story, and puzzle etc. ii) go beyond the mathematics textbooks, iii) not to devote excessively long time on practice of exercises on mathematics, iv) use flash cards, pictures, diagrams, flow charts, graphs, objects etc. for better understanding of mathematics concepts.

***Teaching learning equipments*:** Teacher should collect mathematics text books, mathematics reference books, magazines, mathematics magic, story and puzzle books, project books, books related to history of mathematics and mathematicians etc. Teacher should talk to the students, parents and community members to collect/ prepare mathematical equipments.

***School environment*:** The environment of school plays a vital role in mathematics learning. The environment should develop in such a way that the child may be motivated to learn mathematics. Walls of the class rooms and schools should be designed with mathematical concepts. Innovative ideas on mathematics might be written on the walls. In prayer class also we may read the history of some mathematicians. It is necessary to bright classrooms with display of children's work and other interesting materials.

***Learning corner*:** Basic facilities such as: flash cards, stones, sticks, objects, pictures, cutouts, charts, calendars, playing cards, cartons etc must be available in the class room. Teacher should be prepared/ collected different activities on mathematics and kept these in the learning corner. Teacher should use the learning corner when and where necessary.

***Recreational Activities*:** The recreational activities which are almost ignored in our schools should be given importance as it motivate the students and develop positive attitude among the students. Recreational activities are: organization of mathematics club, mathematics quiz, competition on mental arithmetic etc. Development of question bank (oral, written and performance),activity bank with competencies, answer to Olympiad questions, enrichment and remedial materials.

***Assessment*:** Learners must have opportunities to evaluate their own achievement in practice sessions. When children wrongly answer the questions, do not put them to shame. Master the required competencies before proceed to next competency. Practice sessions definitely must not destroy interest in learning. It is important that pupils enjoy practice sessions devoted to mathematics so that positive attitudes are developed. Avoid physical punishment and the school should be "Punishment free zone".

3.4 LET US SUM UP

Development of abilities for mathematization is the broader aim of mathematics education which includes abilities like problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualization, representation, reasoning and proof, making connections, mathematical communication.

The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems.

The narrow aim of school mathematics is to develop so called 'useful' capabilities, particularly those relating to numeracy- numbers, number operations, measurements, decimals and percentages.

The specific aims of mathematics education in the cognitive, affective and psychomotor domains help to design suitable methods, curriculum, guide to prepare TLMs, prepare evaluation questions etc. Thus it is desirable to write the specific aims in action verbs, pin pointed, short, and in achievable terms.

The vision of school mathematics has been built in the NCF- 2005 expecting every child to learn mathematic with joy perceiving its abstractness, basic structure, the interrelationships, and ways of communication.

Children learn not only from the teacher, but also from interaction with other children, environment around them, nature, materials and things both through action and through language.

The aim is to make learning in school a happy experience breaking the boundary between learning within classrooms and learning outside the classroom using a variety of materials.

Reduce wide gaps between mathematical theory and practice by linking mathematics with life situations.

3.5 MODEL ANSWERS TO CHECK YOUR PROGRESS

E-1 To develop the habits of concentration, self–reliance, power of expression and discovery. To develop thinking and reasoning abilities to solve mathematical problems of daily life.

- To understand and acquainted with the environment and culture.
- To prepare the child for various technical and general future professions.
- To develop in the child the power for invention.

E-2 The specific aims of mathematics education help to design suitable methods, curriculum, guide to prepare TLMs, prepare evaluation questions etc.

E-3 iii ,iv, vi, vii and xi are instructional objectives of mathematics.

E-4 Field trips, mathematical games, puzzles and stories, mathematics magic, linking mathematics with life situation, analysis, representation and interpretation of data, identifying, expressing and explaining patterns, estimation and approximation in solving problems .

E-5 Write 100 with five identical digits: 5x5x5-5x5=100.

3.6 SUGGESTED READINGS AND REFERENCES

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3.7 UNIT- END EXERCISES

1. What difficulties one will face if he does not know A.B.C. of mathematics?
2. Distinguish between educational objectives and teaching objectives.
3. Discuss the aims of teaching mathematics at the elementary level.
4. State the vision of elementary school mathematics.
5. Select ten examples from your surroundings that can be used in your class for teaching mathematics.

Notes

UNIT 4 LEARNER AND LEARNING-CENTRED METHODOLOGIES AT ELEMENTARY LEVEL

Structure

4.0 INTRODUCTION

You have a lot of experiences in teaching mathematics at the elementary level. You might have realised by now that teaching mathematics to young learners at the elementary school learner is not an easy job. It is quite challenging particularly in making mathematics learning more meaningful for the learners. Very often, guided by popular sense that mathematics is a difficult subject, we transfer that feeling to our students. Further, stressing more on textbooks for teaching mathematics, we stress more on rote learning without properly considering their prior knowledge of mathematics, their interest in learning mathematics, learning needs and their learning styles. Such rote learning not only makes understanding of mathematical concepts more difficult but increases fear for mathematics which inhibits further learning of the subject.

Of all school subjects, mathematics has the most systematic and highly organised structure. In addition to this characteristic, the concepts included in the elementary level mathematics are intimately associated with the concrete real life experiences of the learner. If we know how to associate the learning of mathematical concepts using the daily life , experience in the context of the learner thereby activating his/her capabilities to reason, analyse, draw conclusions, then the mathematics learning would be more meaningful, relevant and interesting for the learner. To attain this ability, a teacher needs to be acquainted with the various methods and approaches available for teaching and learning mathematics especially at the elementary level.

This unit focuses on the discussion of learner centred and learning centred methods and approaches of learning and teaching mathematics at the elementary school level. Besides, we have also tried to acquaint you with the traditional methods of teaching mathematics and their continuing relevance.

For the study of this unit and understanding the concepts embedded in it, you need at least 7 (seven) study hours.

4.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Identify the different methods and approaches of teaching and learning mathematics at the elementary level.
- Adopt learning centred approaches and methods of mathematics in your classrooms.
- Organise different activities in the school for making mathematics learning more meaningful, challenging and satisfying.

4.2 METHODS FOR TEACHING AND LEARNING MATHEMATICS

Think for a while regarding what you exactly do in your mathematics class - at the beginning you introduce the concept for drawing attention of the learners towards the topic; then you try to explain that concept through demonstrating different materials, performing activities, or doing such other activities to clarify the concepts making the students to praticipate; and lastly you ask some questions for assessing whether the learners have learnt the concepts as you desired. You follow a proper sequence in your teaching which is usually called a method of teaching. Methods are the style of presentation of content in the classroom so that all students would learn the concepts.

Your style of teaching and the way you present the lesson is always the same. It depends on the nature of the content, the learning style of the students and also depends on the availability of the resources in your classroom. Depending on these factors, you follow different methods for teaching different concepts in mathematics at different times. Here we discuss the methods typically used to teach mathematics in our classrooms.

4.2.1 Inductive-deductive Method

It is perhaps the oldest and the most basic method of teaching as well as learning mathematics. All other methods in mathematics utilise this method in different degrees. This is a combination of two methods of induction and deduction.

Inductive Method: Induction is the form of reasoning in which a general law or principle is derived from a study of particular objects or specific processes. Induction is based on the logic that if something is true for a particular case and is further true for a reasonable adequate number of cases, then it is true for all such cases. Students observe the relationship among such cases, which lead them to guess a common pattern. Thus a formula or the generalisation is arrived at through a process of inductive reasoning. Let us study some examples:

Example 1:

(a) $1^2 = 1$, $3^2 = 9$, $5^2 = 25$, $7^2 = 49$, ………………. where 1,3,5,and 7…. are odd numbers and so also their respective squared numbers 1, 9, 25, 49, …..

(b) $2^2 = 4$, $4^2 = 16$, $6^2 = 36$, $8^2 = 64$ ……………….. where 2, 4, 6, 8,…. are even numbers and so also their respective squared numbers 4, 16, 36, 64,…….

From (a) we get, 'square *of an odd number is odd'*.

And from (b) we get, '*square of an even number is even'*.

Example 2:

1 + 1 = 2; 1 + 3 = 4; 1 + 5 = 6; 3 + 5 = 8; …..where 1, 3, 5, are odd numbers and the result of their addition i.e. 2, 4, 6, 8, ….. are even numbers.

From these additions we can conclude that *addition of two odd numbers is an even number*.

ACTIVITY - 1

Examine whether this generalization is true for (1) addition of three odd numbers, and (2) addition of odd/even number of odd numbers.

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Example 3

1. $a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5 = a^{2+3}$
2. $a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a) = a^7 = a^{3+4}$
3. $a^3 \times a^6 = (a \times a \times a) \times (a \times a \times a \times a \times a \times a) = a^9 = a^{3+6}$ so on.

From these instances we can conclude that

$a^m \times a^n$ = (a × a × ….m times) × (a × a × … n times)

= a × a × … (m + n) times

$= a^{m+n}$

$\therefore$ $\mathbf{a^m \times a^n = a^{m+n}}$

E1. From the Table 4.1 above, what can you conclude about the measures of two adjacent angles?

E2. With appropriate examples, draw inductive conclusions regarding the square of the addition of two real numbers.

Deductive method: Here the learner proceeds from general to particular, abstract to concrete and formula to examples. A preconstructed formula or principles be told to students and they are asked to solve the different relevant problems with the help of the earlier formula. So in this method first you give the relevant formula, principles and ideas to students and explain further its application of the formula to problems. The students in your class come to understand how the formula can be used or applied. For example – when you are going to teach about profit and loss, directly you announced the formula of interest i.e. I = PTR/100 and solve different related problems by using this formula. The students just observe your method of solving and than they memorise this formula for further use.

Deductive approach proceeds from:-

General rule to specific instances

Abstract rules to concrete instances

Deductive approach of teaching follows the steps given below for effective teaching

Clear recognition of the problem

Search for a tentative hypothesis

Formulating of a tentative hypothesis/Choosing the relevant formula for solution.

Solving the problem.

Verification of the result

Example 1: Find $a^2 \times a^{10}$ =?

From the law of indices, it is known that $a^m \times a^n = a^{m+n}$

Hence, $a^2 \times a^{10} = a^{2+10} = a^{12}$ (here m = 2 and n = 10)

Example 2: Find $(102)^2$ = ?

We know that $(a + b)^2 = a^2 + b^2 + 2ab$

$(100+2)^2 = 100^2 + 2^2 + (2 \times 100 \times 2)$ (in this case a = 100 and b = 2)

$= 10000 + 4 + 400 = 10404$

We can also multiply 102 with itself to get the same result (for verification of the correctness of the earlier result)

Inductive and deductive method is the combination of two approaches of teaching mathematics at the elementary level. In essence, the inductive method begins with presentation of specific examples and ends with the formation of generalised principles. In contrast, the deductive approach begins with presentation of generalised principles ends with the generalisation of specific examples.

Inductive method helps the learner in developing the ability to reason by observing common elements in the similar instances and arriving at the generalised statement or rule. Deductive method is all about applying the established rules and formulae in solving various mathematical problems. Almost all the problems in mathematics textbooks can be solved through the application of deductive method.

E3. The method based on the principle of generalisation or establishment of formula/ laws/principle from the observation of concrete examples is the ....method.

E4. Which method focused on application or use of formula directly on problem?

4.2.2 Analytic and Synthetic Method

You have a lot of experiences about different concept in geometry and algebra, where the students proceeds in the logical sequence like- given **A** is true, therefore **B** is true, hence **C** is true. Here **A** is known to be true and the status of **C** is unknown and required to be ascertained as true. Proceeding from A to C is **Synthetic**. But the other way, sometimes student proceeds from unknown to known, i.e. C is true, if B is true; B is true, if A is true. The way of proceeding from unknown C to known A is called **Analytic** method.

In **Analytic method** we break up the unknown problem into simpler parts and then see how these can be recombined to find the solution. So we start with what is to be found out and then think of further steps or possibilities that may connect the unknown built the known and find out the desired result. The nature of the analytic method is that

It leads from conclusion to hypothesis.

It proceeds from unknown to known.

Example. If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d}$

Following analytic method, we start with what is to be proved and proceeded as follows:

If $\frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d}$ is true (we are not sure) then it follows from it that.

$d(ac - b^2) = b(c^2 - 2bd)$ which implies (by simplification of both the sides) that:

$acd - 2\,b^2d = bc^2 - 2b^2d$ would be true, and in turn implies that

$acd = bc^2$ would be true, which again implies that

$ad = bc$ would be true. From this relationship it is implied that

$\frac{a}{b} = \frac{c}{d}$ would be true, which is the given condition.

Using the mathematical symbol '⇒' to read "implies", we can write the above analytic proof in a more concise manner as follows:

$\frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d} \Rightarrow d(ac - 2b^2) = b(c^2 - 2bd)$ (by cross multiplication)

$\Rightarrow$ $cd - 2\,b^2\,d = bc^2 - 2b^2\,d$ (multiplying and simplifying)

$\Rightarrow$ $acd = bc^2$ (by cancelling the equal term '$-2\,b^2d$' from both the sides)

$\Rightarrow$ ad = bc (dividing both sides by 'c' supposed to be a non-zero term)

$\Rightarrow$ $\frac{a}{b} = \frac{c}{d}$, (dividing both the sides by 'bd')

Since, this condition is given to be valid, then $\frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d}$ is also valid as per the above analysis .

Analytic statements are not considered as the statements of proofs for the problem. Rather analysis is considered as the means of discovering the proof. Synthetic method provides the proof of the problem.

In **Synthetic method,** we proceed from what is given to proving what is required. To synthesizes means to place together things that are apart. It starts with the data available or known and connects the same with the conclusion. It is the process of putting together known bits of information to reach the point where unknown information becomes obvious and true.

Let us consider the example given above i.e. if $\frac{a}{b} = \frac{c}{d}$, prove that

$$\frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d}$$

The synthetic proof of this problem runs as follows:

Given $\frac{a}{b} = \frac{c}{d}$ $\Rightarrow ad = bc$ (by cross multiplication)

$\Rightarrow acd = bc^2$ (by multiplying a non-zero quantity '*c*' to both the sides)

$\Rightarrow acd - 2b^2d = bc^2 - 2b^2d$ (adding '$-2b^2d$' to both the sides)

$\Rightarrow d(ac - 2b^2) = b(c^2 - 2bd)$ (taking common elements from both the sides)

$\Rightarrow \frac{ac - 2b^2}{b} = \frac{c^2 - 2bd}{d}$ (dividing both the sides by '*bd*')

This is the type of proof we come across in nearly all mathematics textbooks and literatures. This is precise, logically arranged in proper sequence and easy for reading and communicating.

Analytical proofs look somewhat disorderly which they actually are not. Many are of the opinion that through synthetic method we get the proof, while analytic method provides the way to discover the proof. In that sense, methods of analysis and synthesis in mathematics are complementary to each other.

Method of analysis and synthesis is applied for such problems in mathematics where 'if – then' type of logic is needed (***If*** a triangle is isosceles, ***then*** prove that the measures of the angles opposite to the sides of equal length are equal). In such problems, some conditions (hypotheses) are given and under those conditions some relationships have to be proved. In proving geometric relationships, and algebraic identities and in solving algebraic problems this method can be effectively used.

4.2.3 Project Method

There are number of students in your classroom who are good in solving the problems from the mathematics textbook. You will find most of them are unable to solve the real life problems where the solution remains similar. Take an example, students are familiar to solve the problems on profit and loss from the text book, but they fail to apply the same knowledge during marketing. The reason is the way of teaching mathematics in the classroom. Students are made to spend many hours of the day in learning and repeating subjects from textbooks without understanding their value in daily life. In reality, learning mathematics prepares a child for life by making him live in reality and provide him opportunities where he/she can exercise his/her ability of thinking and skills of doing. Therefore, learning through project is an important aspect for getting real experiences.

The project based learning is a learner-centred method in which the students are challenged to do something by themselves outside the realm of normal class work. Project-based learning is an individual or group activity that goes on over a period of time, resulting in a product, presentation, or performance. A project of mathematics contains rich activities, active participation, freedom to students and correlation with other subjects. In the project based learning, as a teacher the first task is you have identified the area of the project. Then you have to distribute each areas to different groups accordingly the interest of the group. The project based learning followed the following steps.

a. Providing the situations

b. Choosing and purposing

c. Planning for the project

d. Executing the project

e. Judging the project

f. Recording the project

For the grade-VIII students you may assign the project like – running of co-operative bank in your school, laying out a school garden, planning and estimating the construction of a house. For further elaboration of this method you can refer to the Unit 14 in the Paper 3rd of this course.

E5. Identify the basic characteristics of the project in mathematics.

E6. In which method the proof of a geometric theorem proceeds just the reverse way of the proof given in the textbooks?

4.2.4 Problem Solving and Problem Posing

You know solving problems in mathematics is an important skill expected of the learner. It is not always possible to reach the solution of the problem by simple applying the ready made formula. In order to solve these problems, there is need to identify the nature of the problems, collection of information from different sources, analyse and interpret the information to arrive at a solution of the problems. Therefore *problem-solving method* is a process that involves collection, organise, analysis and interpret skills of information for arriving at the solution. You might have experienced that the solution of a problem in mathematics, at least the problems at the elementary school level is considered to be unique, but there are more than one way to reach at the solution. This you can observe in a simple example of addition:

Suppose the students of class II in a school are given to find the results of 75+29 in as many ways as possible. Let us see how many ways it can be done:

I. By direct method: 75 + 29 = 104

II. 75 + 29 = 75 + (30 – 1) = (75 +30) – 1 = 105 – 1 = 104

III. 75 + 29 = 74 + 1 + 29 = 74 + 30 = 104

IV. 75 + 29 = 75 + 25 + 4 = 100 + 4 = 104, so on.

All these processes are correct. Therefore, while we are teaching to solve any mathematics problem, we need to recognise the ways it can be solved. Each student should be made aware of the fact that each mathematics problem can be solved in several ways and they should be encouraged to look for alternative methods for solving any problem. Searching for alternative solutions requires reflective and creative thinking abilities on the part of the learners. The main objective of problem-solving method is, therefore, to stimulate the reflective and creative thinking of the learners. In order to solve a problem in mathematics, a learner needs to proceed along the following steps:

a. *Identifying the problem*: The students should be able to identify the problem before they attempt to solve it.

b. *Defining the problem*: sometimes restarting a problem in the student's own words; help him to understand the problem in terms of what is given what are to find out in a problem?

c. *Collections of relevant information*: Here the students will collect related information that is needed to solve the problem. Recall of previous learned knowledge, facts, skills, theorems, and processes etc, the students may learn to ask what I know that is related to this problem. For an example, in height and distance problem, one needs to recollect the trigonometric ratio.

d. *Formulating tentative hypothesis*: The focus of this stage is on hypothesising-searching for a tentative solution to the problem. For example, when students are going to find out the total surface area of cone, they may formulate the hypothesis as total surface area is the sum of the curved surface and the base area of the cone.

e. *Testing the hypothesis*: Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternative hypothesis and proceed.

f. *Construct physical models*: Some problems need physical model for finding the solution. For example, how many 1×1 squares are there in 8×8 chess board? Children may be provided with chess board for finding the solution of the above question.

g. *Verification of the result*: At last the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalisation and apply it to their daily life.

The details of problem solving method have been discussed in the Unit 3 of the Paper 3 of this course.

Problem posing is closely associated with the problem solving method. Problem posing involves generating new problems and questions to explore about a given situation, as well as reformulating a problem during the course of solving the problem related to it. Teachers can help to developing this habit by understanding the children's thinking processes and developing these processes using generative questions. The problem posing method involves developing problem posing as an instructional intervention to improve problem solving skills and to improve disposition towards solving. Problem posing is an indicator of learning that takes place.

When we encourage children to be problem posers, we are inviting them to do what mathematicians do — that is, to look closely, seek patterns, offer conjectures, and set out on paths that are not clearly marked. In the process of their investigations, mathematicians also develop attitudes about learning, such as perseverance, willingness to revise their thinking, and appreciation for the value of risk taking.

Let us, as an example, consider the statement $4 \times 5 = 20$.

The first step of problem posing is to look closely or observe the statement critically. In the above statement following are some of the observations we can make.

There are two multipliers.

The two multipliers are two consecutive natural numbers.

One of the multipliers is even and other is odd.

One is a multiple of 2 and other is multiple of 5.

The product is 4 more than a square number (16) and 5 less than another square number (25).

The multipliers are consecutive counting numbers.

The difference between the multipliers is 1.

Notes

After making the observation on the statement $4 \times 5 = 20$, what possible problems can be posed? Some problems posed here as exemplars:

1. Do we always get an even product when multiplying an odd number by an even number?
2. What do we find if we multiply an odd number by an odd number? An even number by an even number? What if we multiplied three odd numbers or three even numbers?
3. What if we continued to multiply by multiples of 2 and multiples of 5? What patterns might we see?
4. What if we tried using multipliers that are the same to make 20? Is this result possible? What products are possible using multipliers that are the same?
5. What if we tried adding two numbers to equal 20? How many ways could we do so? What do we notice about odd and even numbers when adding to make 20?
6. Why, when we add an odd and an even number, do we get an odd number, but when we multiply an odd number by an even number we get an even number?

What are the benefits of problem posing for learning?

It develops the spirit of inquiry. The more we observe, the more we want to find out.

It leads the learner into unknown territory.

It requires and promotes reflective thinking especially during posing the problems.

It supports learners in asking the perennial question that mathematicians pose: Is this always true? That is, did this relationship occur fortuitously, or does a pattern lurk behind these numbers?

The other benefit is that problem posing involves searching for patterns.

Uncovering patterns is certainly joyful, but even more rewarding is discovering why those patterns are occurring. Let's see what we notice about our original problem: $4\times 5 = 20$. We see that 20 is 4 away from the nearest smaller square, 16, and is 5 away from the nearest larger square, 25. Why? We notice that 4×5 is $4 \times (4+1)$ or $(5-1) \times 5$.

E7. Pose some problems relating to the geometric theorem "Sum of the lengths of any two sides of a plane triangle is greater than that of the third side" along with the points of observation.

4.3 LEARNING- CENTRED APPROACHES OF TEACHING MATHEMATICS

In your mathematics class, what do you usually do? Since, mathematics is considered as a difficult yet an important subject, you spend most of the time in the classroom explaining the concepts, formulae, diagrams, solving difficult problems, asking and answering questions etc. In brief you are extremely busy in instructing and directing students who are comparatively less active. Your students listen carefully your teaching and take down notes of the relevant points. In this type of learning environment, where the teacher is more engaged in explaining, students have limited opportunity to ask questions or may be uncomfortable asking a single question in the classroom. There is very little scope for the students to be involved in the classroom discussion. The methods of teaching mathematics that we discussed in the previous section are mostly teacher centered. On the other hand in the learning- centred approaches the focus is how students will construct their own knowledge on basis of their prior experiences where the teacher is truly a facilitator of learning. In this section, we shall be discussing three such methods or approaches.

4.3.1 5E's Learning Model

In this model of learning, students learn in five sequential phases i.e. Engagement-Exploration-Explanation-Elaboration-Evaluation.

I. Engagement phase: In the *Engagement Phase*, students are engaged in the classroom through different learning tasks. This learning task may be an activity, showing any surprising event, peculiar examples etc, where students will get an opportunity to relate their previous knowledge with the existing ideas. Your job in this phase is to identify the earlier knowledge of students and their misconceptions/ alternative conceptions related to the concepts they are going to learn. The *engagement* component in the 5E'S learning model is intended to capture students' attention, get students thinking about the subject matter, raise questions in students' minds, stimulate thinking, and access prior knowledge. As for example, suppose you are going to teach the **concept of fraction** to Grade- VI students. You might engage your students in an activity, as suggested below, which involves their prior experience related to learn fraction.

Tasks I: Distribute a few pieces of paper (circular, rectangular shape) and thread to all students and ask them to divide it into two parts. The students may divide the paper/ thread either in equal or unequal parts.

Task II: Make a presentation with paper sheets of same and different colour with geometrical figures of circle and rectangle.

One part of each figure (Two equal and unequal parts) is coloured (Fig.4.1)

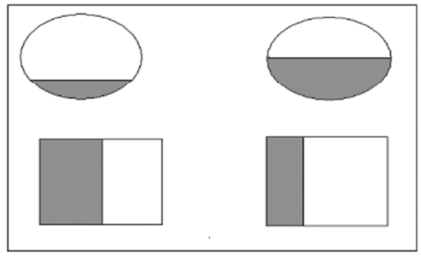
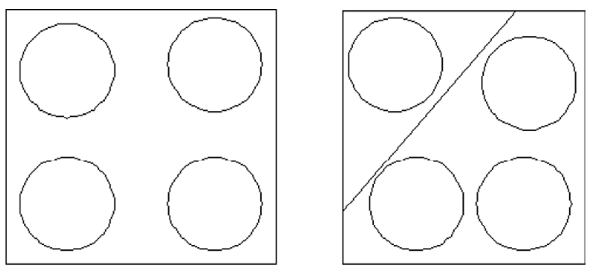


Fig. 4.1 Fraction of single figure

One sheet showing equal division of objects and another showing unequal partition of a collection of objects (Fig. 4.2)



(A) (B)

Fig. 4.2 Fraction of collection of figures

While demonstrating the figures, you can ask the students to clarify the concepts associated with 'equal parts of a single object or collection of objects' to ascertain their previous experiences.

Such conversation in the engagement phase will enable the learners to construct their knowledge on fraction rather than directly receiving from the teacher.

II. Exploration Phase: In the *Exploration* phase the students have an opportunity to get directly involved with the phenomena and materials. Involving themselves in these activities they develop a grounding of experiences with the phenomenon. One of important characteristics of this phase is students' collaboration (i.e. group work). As they work together in teams, students build a base of common experience which assists them in the process of sharing and communicating. You will act as facilitator, providing materials and guiding the students to focus. The student's inquiry process drives the learning during an exploration. From the above tasks in the engagement phase students will explore individually and after they work in group, they come to the conclusion that, **fraction** is a part of the whole**.** When the whole is divided into two equal parts, then each part is called one half of the whole. This is expressed /written as one by two (1/2) or one upon two.

III. Explanation Phase: The third stage, *Explanation* is the point at which the learner begins to put the abstract experience and clarify their mis-conception through discussion in the classroom. You will explain the concept only after students have got the common experiences through collaboration. In this phase your role is to explain, that does not mean you will not involve the students in the discussion process. The degree of explanation depends on the understanding and misunderstanding of the students.

IV. **Elaboration Phase:** The E*laboration* phase of this learning cycle provides an opportunity for students to apply their knowledge to new situations, which may include raising new questions and hypotheses to explore. The students expand and apply on the concepts they have learned from the earlier three phases, and make connections to other related concepts, and apply their understanding to real world around them.

V. **Evaluation Phase:** Evaluation, the fifth 'E', is an on-going diagnostic process that allows you to determine, whether the learner has attained understanding of concepts and knowledge. You may use different techniques of assessment in the classroom like portfolio, assignment, observation, concept mapping, peer assessment etc.

E8**.** In which phase of 5E'S learning model, students get chance to reflect on their knowledge.

4.3.2 Interpretation Construction (Icon) Design Model

Learning in this model comprises of seven steps and start from the learner's observation.

1. ***Observation*** is the key aspects of this model, where learners are made to observe the elements and situation related to the problem before proceeding to evolve a solution.

2. ***Contextualization:*** After observing the situation learners try to contextualise the situation. That is the learners relate the observed situation and elements of the problem to be solved to their previous ideas/ experiences/knowledge.

3. ***Cognitive apprenticeship:*** Learners are encouraged to exercise their minds though different brainstorming activities by the teacher. As a teacher, you need to guide them how to analyse and interpret the problem at this stage. You will find your students are having several alternative conceptions or misconceptions.

4. ***Collaboration***: Learners form group to work on the task. In the collaboration stage**,** learners discuss freely about their alternative conceptions/misconceptions and are able to communicate with their peers. As a teacher, your work is to guide each group and at the same time you also act as a co-learner of each group.

5. ***Interpretation & Construction***: The learners analyze their constructed knowledge through argumentation, discussion and validation and generate their own interpretation.

6. ***Multiple interpretations:*** As learners have a lot of flexibility during the learning process, they are able to interpret the knowledge in different ways and different manner and form several possible interpretations of the problem situation as well as problem solution.

7. ***Multiple manifestations***: The learners try to apply various interpretations one by one for the problem solution and thus acquire multiple solutions to the problem. Further they also gain multiple manifestation of the same interpretation.

You can realise your role, as a teacher, in such a method which require total involvement of the learners along with you in a pursuit of innovation. Your major role is to facilitate the group interaction and keep the participants focussed on the problem. This requires a lot of imagination and patience on your part to mobilize the learners' capabilities, their willingness and enthusiasm and over all their pool of previous knowledge enabling them for multiple interpretation of the problem and going for multiple manifestations.

This method when applied in mathematics teaching – learning processes in the classroom situation, helps both the learners and the teacher in successfully formulating multiple ways of solving a problem which was thought to be possessing only one correct method of solution.

4.3.3 Concept Mapping

Students have learnt different concepts in mathematics, but many times you will find they treat the concepts they have learnt as disjointed and isolated facts and they are unable to interlink the relationships between these concepts. You also know that no concept in mathematics is isolated; a particular concept of mathematics is interlinked with different branches of mathematics and with other subject like science and social science in different ways and different manners. As for example, a concept map on quadrilateral is given below:

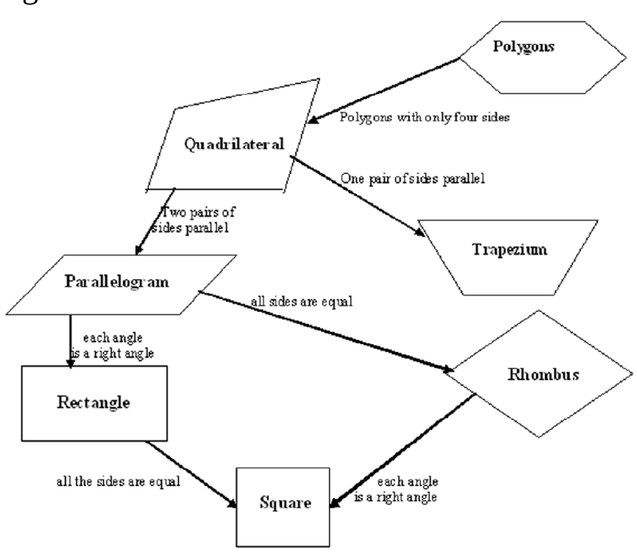


Fig. 4.3 Concept map of quadrilaterals

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There are different ways/ types of concept maps can be formed on a particular concept. The above concept can be developed in different maps, depending upon the number of sub-concept and linking words you want to include. Therefore, the number of connections and depth of understanding can be assessed by the number of linking lines and used by the student's concept map. Hence a concept map provides a concrete record of the connections perceived by the students, and thus it indicates how the student's knowledge is organised and interconnected. More specifically, concept mapping can furnish valuable insight into the depth of students' understanding because it reflects the accuracy and strength of their connections. Even Venn Diagram of some concepts can play a role of concept mapping

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E9. In which step of ICON design model, students relate their previous knowledge.

E10. Why arrow mark is used in concept map?

4.3.4 Activity Based

You might have observed in your classroom, that students have shown interest when you are performing or they themselves are performing any activity/activities in the classroom. The reason behind is that a young child loves to use his/her different sense organs during learning. Activity based learning focuses use of these sense organs and learning should be based on doing some hands-on experiments and activities. The idea of activity-based learning is rooted in the common notion that children are active learners rather than passive recipients of information. If child is provided the opportunity to explore his/her own and provided an optimum learning environment then the learning becomes joyful and long-lasting. The key feature of the activity approach is that it uses child-friendly educational aids to foster self-learning and allows a child to study according to his/her aptitude and skill. At school level in mathematics the activity/activities may be in the form of game, puzzle, worksheet, paper folding/paper cutting, concept mapping of mathematical modelling etc.

Suppose you are going to teach the algebraic identity: $(a+b)^2 = a^2+2ab+b^2$. You may prove it numerically on the black board, but when we talk about to learn the same algebraic identity using activity, students may prepare a model using thermocol sheet, adhesive, thermocol cutter, glaze paper and sketch pen, during their preparation you may guide them and you demonstrate it and prove this identity.

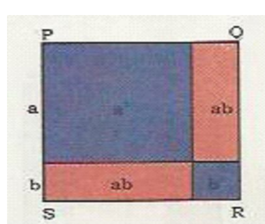

Fig. 4.4 Model for $(a + b)^2$

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Consider another example. Suppose you are going to teach the properties of triangle to grade VI students. You may demonstrate an activity, by taking three sticks and prove the properties like sum of any two sides of any triangle is greater than the third side. By using the paper folding and paper cutting activity you may also prove the other properties like sum of angles of any triangle is equal to 180 degree. If you will be followed such activity based learning in your classroom, you will find how students are curious and enjoying the activities. In unit 4 of the paper3 of this course, you might have already discussed the details of activity-based approaches of teaching and learning which can be equally applicable for mathematics.

Experiential Learning is an approach to learning in which participants engage in an activity, reflect on the activity critically, and obtain useful insight and learning. Learning which is developed experientially is "owned" by the learner and becomes an effective and integral aspect of behavioral change. Skill development occurs through Experiential Learning.

Fig. 4.5 Experiential Learning Cycle

The Experiential Learning Cycle includes five sequential steps, or stages. The steps are as follows:

Experiencing: (This is the initial stage of the cycle): Almost any activity that involves self-assessment or interpersonal interaction may be used as the "doing" part of experiential learning.

Publishing: After participants have experienced an activity, they are ready to share or publish what they observed and how they felt during that experience.

Processing: (This is the pivotal step in the experiential learning cycle). This step, referred to as the group dynamics stage, includes systematic examination of shared experiences by the members of the group.

Generalizing: In this stage, the members of the group begin to focus on their awareness of situations in their personal or work lives that are similar to those they experienced in the group.

Applying: In this final stage, the facilitator helps participants apply generalizations to actual situations in which they are involved.

Suppose you are going to teach the concept of rate of interest in mathematics. Student needs to get some experience, really how interest increases time to time. Therefore to learn such concepts, you may assign the students to observe the passbook of your parents of any bank. Students will understand and better way to apply the idea of interest and rate of interest in different situations if they learn it through experience.

4.4 MAKING MATHEMATICS LEARNING MORE CHALLENGING AND SATISFYING

One of key objectives of teaching mathematics is to make learning more challenging and satisfying. That means, learning mathematics have to create curiosity among students and place them in a challenging situation, where they can achieve their level of satisfaction. At the same time, learning mathematics must have to give the pleasure during the leisure period of learners, and also it should reduce the anxiety and stress to learn mathematics. You might have observed that students are failing in mathematics as they develop fear before they start to learn. Therefore it is your duty to make mathematics learning more challenging and satisfying, so that learners will be enhance their creative abilities and show the positive attitude to learn mathematics. Let us discuss how learning mathematics helps to develop learners' creative abilities and the use of mathematics laboratory and library.

4.4.1 Development of Learners' Creative Abilities

You might have observed in your classroom that some of the students' performance is different from others. Their ways of solving a mathematical problem or developing any kind of activities in mathematics are unique and innovative. These abilities of the learners called as creativity or creative abilities. The important task of teaching mathematics is how to develop such abilities among the learners. The development of learners' creative abilities more or less depends on the nature of learning task and the approaches followed by the teacher in the classroom. Let us identify the nature of the learning task that impacts on students' creativity

Activity based: Tasks would be designed in such a manner that the students are attracted to it and participate in it spontaneously.

Challenging: The task should neither be too easy nor too difficult but must be mentally challenging for the learner so that he/she attends to it employing his/her full potential and solving the task is mentally satisfying.

Divergent solutions: Unlike most of the mathematical problems each of which has only correct answer, tasks encouraging creativity need to have several possible solutions. This encourages learners to search for innovative solutions., *Logical and problem oriented:* Unlike problems in other areas, mathematical problems have a distinct logical structure and all mathematical tasks are problem oriented. Once the learner becomes familiar with the logical structure, he/she tries to decipher the logic and in the process tries to employ several innovative processes to solve the problem consistent with the logical structure. The problem orientation of the tasks poses challenging situations for the learner to evolve new approaches for solution.

Pictorial/graphic representation: Representing mathematical data and relations in various pictorial and graphic forms encourages creative talents.

Similarly the approaches of learning must have to address the following for the development of creativity among the learners

Recognition of alternative ideas/methods of solutions proposed by students.

Collaboration- Both students and teachers search for alternatives.

More scope for brainstorming and reflective thinking.

Encouragement for divergent thinking.

More scope for problem posing and problem solving.

Freedom to students for questioning and expressing ideas.

More scope and freedom for fluent expression and elaboration of ideas.

Motivation and accepting views/suggestions.

Active Learning and process based assessment.

4.4.2 USe of Mathematics Laboratory and Library

In your classroom, suppose, while teaching the area of the circle to grade-VII students, you just give the formula of the area of circle and solve the numbers of problem on black-board in a routine way. This way of teaching is product based on product, focuses on computational skill among the learners. Students may not come to know why and how the formula of circle is ? r2. Therefore process of learning mathematics is vital for the construction of knowledge and use of mathematics laboratory and library facilitate the process based learning. Learning mathematics is both a creative and explorative process, and at the school level use of child resources in the process is more important. Every student of mathematics needs to learn the mathematics process. The best way to learn the process is to practice it. However, in the classroom students are given little chance to experience the full process of creating and exploring mathematics. Instead, they are taught merely about the products of the process. Therefore the best way to learn mathematics is to use mathematics laboratory as it can act like a concomitant between teacher and students and provides an opportunity to understand and discover the beauty, importance and relevance of mathematics as a discipline. It can be expected to enhance the pupil's understanding of the subject as taught at the school and can also provide a pleasure to learn mathematics.

A Mathematics Laboratory is a place where some of the mathematical activities are carried out and the students get hands-on experience for new innovations. Further Mathematics Laboratory can foster mathematical awareness, skill building, positive attitude and learning by doing in different branches of mathematics. It is the place where students can learn certain concepts using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. Maths Lab also popularizes mathematics.

In your school, you might have on maximum students are familiar with their text book and they think that text book is the ultimate resources to learn mathematics. Besides the text book , now different journals in mathematics, magazines, reference books, CDs are regularly available and they contain the innovative ideas, experimentations, critical use of formula and life story of different mathematicians. Students are needed to learn these materials regularly to get different ideas in the world of mathematics. Therefore use of mathematics library is an important aspect in the process based learning. As a teacher you also used the library and motivated your students to learn from mathematics library.

Mathematics Library is an important place of resources for collection, dissemination of mathematical concepts, themes, story, references, articles puzzles and games.

4.5 LET US SUM UP

Methods are the ways/styles of the transaction of content in the classroom for effective learning.

Inductive method based on the principle of generalisation from concrete instances, in which students are encourages to establish the facts/laws/principle/formula from their observation.

Deductive method based on the principle of the application of facts/law of inferences/principle/formula in solving different problems.

Problem solving and problem posing are close to each other. Problem-solving is a process to arrive at the solution, whereas problem posing involves generating new problems and questions to explore about a given situation, as well as reformulating a problem during the course of solving the problem related to it.

Project is a learner-centred method in which the students are challenged to do something by themselves out side the realm of normal class work.

In the learning- centred approaches the focus is how students construct their own knowledge on the basis of their prior experience.

In 5E's model of learning, students learn five different phases i.e. Engagement-Exploration-Explanation-Elaboration-Evaluation.

A concept map provides a concrete record of the connections perceived by he students, and thus it indicate how the student's knowledge is organised and interconnected.

Activity based learning focuses use of these sense organs and learning should be based on doing some hands-on experiments and activities.

Experiential Learning is an approach to learning in which participants engage in an activity, reflect on the activity critically, and obtain useful insight and learning.

A Mathematics Laboratory is a place where some of the mathematical activities are carried out and the students get hands-on experience for new innovations.

Mathematics Library is an important place of resources for collection, dissemination of mathematical concepts, themes, story, references, articles puzzles and games.

4.6 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. The sum of two adjacent angles of the two intersecting lines is 180^0.

E2. Give exemplars of different types of real numbers for induction. Conclusion would be "The sum of two real numbers is a real number".

E3. Inductive.

E4. Deductive method

E5. Project should contain rich experiences, activities, need collaboration

E6. Analytic method

E7. Transformation of number and sign, simplification etc.

E8. Elaboration

E9. Contextualisation

E10. For linking the concepts

4.7 SUGGESTED READINGS AND REFERENCES

Bransford, J.D., Brown, A.L. & Cocking, R.R. (2000). *How People Learn*. Washington DC: National Academy Press.

Wood, T., Cobb, P. & Yackel, E. (1995). Reflections on Learning and Teaching Mathematics in

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Elementary School. In L. P. Steffe & J. Gale (Eds), *Constructivism in Education*. Hillsdale: Lawrence Erlbaum Associates.

4.8 UNIT- END EXERCISES

1. Select three concepts in geometry at the elementary level describe how you can teach those using inductive method.
2. Select one topic from elementary class mathematics; develop a plan to teach such topic by using both the inductive and deductive method.
3. Identify different tasks on properties of triangle through which you will engage your students in activity- based learning.
4. Prepare a lesson plan on any topic from grade-VII mathematics using 5E'S learning model.